Militarization, Negotiations, and Conflict

Adam Meirowitz*, Kristopher Ramsay†, Noam Reich‡

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Abstract

We situate a growing theoretical literature on strategic militarization in a family of connected parsimonious models in order to flesh out how key institutional or environmental features impact the equilibrium level of arming and risk of fighting. Key comparisons allow us to assess the importance of negotiation dynamics and whether arming decisions are hidden or public. Through the comparison of these models, we help clarify how several structural features interact. We also trace out the central logic behind deterrence, signaling and preventative fighting. A key focus is on the economic and social repercussions of these strategies, and an assessment of what features in the environment are central to the emergence of various forms of behavior. Understanding these behaviors is crucial for scholars and policymakers navigating the complexities of international security and military strategy, and the ability to support a broad range of circumstances in a simple family of models helps to clarify how primitive assumptions relate to equilibrium phenomena.

1 Introduction

For all the attention that the decision to go to war receives, far less attention has been given to the impact of militarization policies on crises, negotiations, and the probability of open warfare. Militarization, the acquisition of arms and mobilization of armies, is in most cases a prerequisite for countries to be able to go to war. In fact, the flow of crises is often determined by a state's needs to militarize, with World War I's mobilization timetables being the most extreme example.

Understanding the incentives for militarization and how arming choices impact decisions about negotiations and warfighting is crucial for analyzing international relations and conflict. In this chapter, we present a survey describing the state of our theoretical

*Damon Wells Professor of Political Science, Yale University
†William P. Boswell Professor of World Politics of Peace and War, Princeton University
‡Postdoctoral Fellow, NYU-Abu Dhabi

understanding of these incentives and explore how features of the strategic interaction between states during a crisis affect militarization decisions.[1]

There are generally two channels by which militarization decisions affect conflict. First, militarization can impact the decisions to use force, influencing the probability of war. A state may refrain from fighting if it thinks its opponent has militarized, and it knows that it has not. Second, because militarization alters the willingness to fight, it can affect the terms of proposed settlements, shaping the outcomes of negotiations. Thus, in most work, the bottom line is an examination of how militarization, and features that shape militarization decisions, impact the odds of conflict and the terms of settlement.

One important environmental feature that influences militarization incentives is its observability. The degree to which other countries can observe a nation's military buildup can have significant implications for negotiations and the likelihood of war. If militarization is observable, it can serve as a deterrent to potential aggressors. On the other hand, if it is difficult to observe, it can create uncertainty, potentially increasing the risk of war.

Scholars also consider arming to be a choice made in settings where participants face real uncertainty about each other's intentions. They see arming as a possible form of signaling. By building up its military capabilities, a country can signal its resolve to other nations. This can influence the behavior of other countries in negotiations and potentially deter aggression. However, signaling through militarization can also be risky, as it may provoke other countries to respond with their own military buildups.

In this chapter, we will explore these and other aspects of militarization in detail. We will draw on the existing literature to provide an overview of the topic and offer insights into how militarization affects negotiations and the incidence of warfighting. Through our analysis, we aim to provide a more in-depth understanding of this complex and important issue and provide future scholars with an overview of the types of models that have been used to explore this topic.

The chapter is organized around a central model of strategic interaction. To capture the results of complex models in a concise and accessible fashion, we develop a simple two-player game in which there is a militarization decision and then a crisis in which war may occur. States ultimately care about a resource or issue, which in some models can be

[1] This review focuses on recent work that has connected strategic arming and the bargaining model of war, building on ideas exemplified by the classic analysis of Richardson (1960) and Jervis (1978). For reviews of these earlier works see Intriligator and Brito (1976) and Downs, Rocke and Siverson (1985). These reviews primarily focus on Richardson style models, classic 2 by 2 normal form games in their analysis of arms races, and empirical studies of arms races. While important, they do not directly connect to the contemporary international relations literature, which typically places war, and consequently arms races, within the frameworks that interpret war as a form of international bargaining occurring in the shadow of potential conflict. For a discussion of the empirical literature, also see Glaser (2000).

divided through a settlement or by capitulating to demands. If war occurs, we abstract from the warfighting process and assume the prize is awarded probabilistically to the winner; warfighting costs may be accrued. Through various modifications to our simple game, we can present stylized versions of strategic incentives that form the basis of much of the literature. This allows one to get a handle on the richness of strategic interactions and tradeoffs and provides some sense of how structural circumstances and modeling assumptions impact the kinds of behavior that may emerge in equilibrium.

The goal of this chapter is to help researchers appreciate the key strategic forces and tradeoffs involved in militarization decisions through the development of a series of models. We purposely abstract away from certain details in published work, highlighting only a subset of the findings in key publications, and we do not claim to exhaustively review all the relevant and important research. That is to say, our chapter is itself a model review, we abstract away from some issues that have been raised by serious scholars to paint a somewhat easier to understand and coherent picture of the state of scholarship.

To be sure, academic research on strategic militarization that develops models is only a small subset of the body of research on militarization strategies. That being said, our focus here is on this subset as a large share of the broader, often empirical, literature contains core ideas or arguments that are derived from other work employing game-theoretic models.

2 Basics of Strategic Arming

We begin by presenting a simple framework using a very basic model we will modify in several ways. Consider the situation faced by two countries in a potential dispute that could lead to war. Country A is the leader, and to begin we will treat their level of military capacity as fixed. Country B is a rising state, and our goal initially is to understand when she might choose to militarize.

To start, militarization is treated as a binary choice by B. She may pay a cost k to become stronger, or she may retain her current level of military capacity. In all variants of the model, there are two kinds of outcomes. The interaction between A and B may end peacefully, in which case a resource of size one is efficiently split between the two players. In a peaceful outcome, let x denote the share to A and $1 - x$ denote the share to B. Alternatively, the interaction may end in war.

The payoffs to war depend on the militarization decision of B. The relative strength of B impacts the odds that B wins. To capture the effect of arming, assume the probability that B wins when they are unarmed is w, and s when they are armed, with $w < 1/2 < s$.

In the sequel, the arming choices of both A and B will jointly determine war payoffs.

As is standard in the literature, assume that war-fighting is costly. Let θ denote the destructiveness of war, where $(1-\theta)$ is the size of the prize after war. In particular, we assume that the payoffs from fighting to A, B are given by

$$\{u_A(war), u_B(war)\} = \begin{cases} \{(1-s)(1-\theta), s(1-\theta)\} & \text{if } B \text{ arms} \\ \{(1-w)(1-\theta), w(1-\theta)\} & \text{if } B \text{ does not arm} \end{cases}$$

Under this construction, war is a lottery that assigns the reduced prize to country A with probability $1-p$ and to B with probability p, where p depends on arming strategies.

Warfighting may be a more complicated process, but at the time of choosing to fight, the option is evaluated at its expected value. We are assuming that actors are risk-neutral, though the qualitative nature of the trade-offs countries face are not significantly affected if they are risk-averse in these models.

As we will see, an important condition for determining whether arming can be rationalized is encapsulated in the following assumption:

Assumption 1. *For the rising state, the cost of arming is smaller than the difference in expected war payoffs between being strong and weak.*

$$(1-\theta)(s-w) > k.$$

If this condition fails, then the cost of arming simply outweighs the gain from arming that a state can obtain on the battlefield. In the class of models considered here, the gains that a state can generate in a peaceful settlement are tied to what she could gain from fighting and so, when Assumption 1 fails, the cost is decisive and selecting not to arm is obvious.

We now build on these fundamentals to analyze a series of models aimed at showing how the observability of militarization and the nature of crisis negotiations, as well as assumptions about the sequence of interactions, shape militarization incentives and the relationship between arming and fighting.

We will consider the following environments.

- Public arming with a fight, not fight decision
- Public arming followed by bargaining
- Repeated public arming followed by bargaining
- Secret arming followed by bargaining

- Simultaneous public arming with bargaining
- Simultaneous hidden arming with bargaining
- Potentially signaling strength and costs with bargaining
- Preemptive attack of a state that can arm

As mentioned, the literature in general, and our string of models in particular, assume that countries care primarily about their share of the resources and possible costs and benefits from fighting. Moving from player objectives to measures of welfare, or socially relevant outcomes, the key metrics tend to be the probability of war, $Pr(war)$ and, to the extent that militarization typically involves costs, the probability that B arms, $PA = Pr(m_B = 1)$. Thus, whenever possible, we will tie our analysis back to these measures.

3 Public Arming

To start, consider a simple game in which first B decides to arm, then, after observing this choice A and B decides whether to initiate conflict or maintain a fixed status quo split $(x, 1-x)$. This model focuses attention on arming incentives alone, but can also apply to situations where an issue has a limited number of feasible peaceful solutions.

If any country chooses to fight, there is a war. Otherwise, there is peace that divides the benefit of the resource between the two countries at $(x, 1-x)$. For any fixed parameterization of the exogenous split, x, the payoffs to the conflict game are shown in Figure 1.

A \ B	$Fight$	$\neg Fight$
$Fight$	$(1-s)(1-\theta)$ $s(1-\theta)$	$(1-s)(1-\theta)$ $s(1-\theta)$
$\neg Fight$	$(1-s)(1-\theta)$ $s(1-\theta)$	x $1-x$

(a) B arms

A \ B	$Fight$	$\neg Fight$
$Fight$	$(1-w)(1-\theta)$ $w(1-\theta)$	$(1-w)(1-\theta)$ $w(1-\theta)$
$\neg Fight$	$(1-w)(1-\theta)$ $w(1-\theta)$	x $1-x$

(b) B does not arm

Figure 1. Two simultaneous 2x2 games with payoffs

Given arming choices, we see that, for A, not fighting weakly dominates fighting if $u_A(war) < x$, and fighting weakly dominates not fighting if the inequality goes the other way. Similarly, we can see when fighting or not fighting are weakly dominated for B given her arming choices. We focus on equilbria in which warfigthing decisions are not weakly dominated.

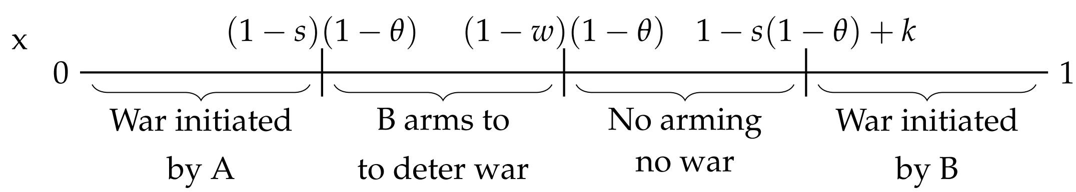

Figure 2. Arming Without Bargaining: When countries cannot bargain, arming can be used to deter a rival from going to war.

Moving up to the choice to militarize by B there are two potential benefits to arming. First, if A is going to initiate conflict independent of B's strength, then the benefit of arming is captured by $(1-\theta)(s-w)$. In order for A to initiate conflict independent of B's strength, we must have $(1-s)(1-\theta) \geq x$

Thus, arming to fight is an equilibrium phenomenon if two conditions hold:

$$(1-s)(1-\theta) \geq x \text{ and,}$$
$$(1-\theta)(s-w) \geq k.$$

The other potential benefit occurs when arming deters conflict so that $(1-w)(1-\theta) \geq x \geq (1-s)(1-\theta)$. The benefit to B is then $1-x-(1-w)(1-\theta)$. Thus, arming to deter is an equilibrium phenomenon if two conditions hold:

$$(1-w)(1-\theta) \geq x \geq (1-s)(1-\theta) \text{ and,}$$
$$1-x-(1-w)(1-\theta) \geq k.$$

A complete characterization of the equilibrium in this game is straightforward.

Proposition 1. *In the generically unique Nash equilibrium in weakly undominated strategies,*[2] *a tranquil path of play (no war, no arming) occurs whenever* $(1-w)\theta < x < 1-s\theta+k$. *A deterrent path (arming, no war) that features arming but no war occurs whenever* $(1-s)(1-\theta) < x < (1-w)(1-\theta)$. *A warfighting path (arming, war) occurs whenever* $x < (1-s)(1-\theta)$ *or* $x > 1-s(1-\theta)+k$.

[2] The equilibrium is not unique only for the knife-edged set of parameters where some of the above inequalities hold weakly.

3.1 Public Arming with Bargaining

We now consider a richer environment where country A can make a credible offer to split the resource through a simple take-it-or-leave-it offer. Though the literature considers more complicated bargaining environments (e.g., Powell 1996; Fey and Ramsay 2011; Bas and Coe 2012), most of the key insights we emphasize here can be achieved within this simple and commonly employed modeling strategy.

Early work focused on cases with limited divisibility, or commitment problems. More recent work tends to focus on settings where endogenous compromise is possible, and so the assumption that states can commit to splits is somewhat standard. Our goal here is to show how the incentive to arm and the probability of a peaceful settlement depend on the ability of players to smoothly divide the pie, and so we focus in on the case where this form of commitment is easy and contrast it with the previous model where no splits are possible.

We begin by allocating proposal rights entirely to A. Explicitly, the game form involves a public decision to arm (or not) by B, as in the first model, and then A makes a take-it-or-leave-it offer $(x, 1-x)$ to split the pie. B then accepts or rejects the offer. A rejection leads to the war payoffs described above, and acceptance leads to payoffs x to A and $1-x$ to B.

In any subgame perfect equilibrium, A must make the offer which makes B indifferent between fighting and peaceful settlement at $1-x$ and B must resolve her indifference by accepting this offer. In particular, following arming, the offer to B is $(1-\theta)s$ and following no arming, the offer to B is $(1-\theta)w$. Moving up to the arming decision, B will arm if and only if the gain in war payoffs from arming is sufficient to offset the cost of arming. This is our Assumption 1 above. It is important to note that this condition is relevant even though there is no risk of actually fighting in this model. So here the path of play mirrors what we called a deterrence equilibrium above–arming but not fighting. Here, however, it is less clear that we should call this deterrence behavior because if B had not armed, there would have also been peace, just with a different agreement.

Proposition 2. *Under Assumption 1, there is a unique subgame perfect equilibrium to the bargaining game with public arming, where the path of play involves arming and then the acceptance of the offer,* $(1-(1-\theta)s, (1-\theta)s))$

A few comments are in order. First, with strategic militarization some friction, like an inability to commit to alternative splits of the pie or, as we will see, private information is needed to generate a risk of inefficient war. But we also see from the baseline model that the incidence of warfighting is not necessary for arming to occur. In this model, militarization serves the role of an investment that raises the share of the pie that B can legitimately expect (and obtain through equilibrium in the bargaining game).

Interestingly, the ability to commit to arming before bargaining is not necessary. Even if the arming decision occurred after a bargain was rejected, but before a war began, by Assumption 1 B would choose to arm. In this case, the credible threat to arm would be sufficient to receive the better distribution of benefits for B. Of course, whether it is reasonable to think that arming can occur quickly enough to make a model with bargaining followed by arming, followed by fighting reasonable is an open question. Finally, here the path of play involves arming and no fighting, which might seem like an example of deterrence, but the equilibrium reveals that absent arming we would still not see fighting. We want to emphasize that the equilibrium analysis illustrates why one should be cautious in drawing conclusions about the effects of arming from observations of just a path-of-play. One needs to have in mind an equilibrium to understanding how militarization impacts things like the incidence of war.

Countries may sometimes find it efficient to arm others. Qiu (2022) studies a model of arming in which a country can choose to split an exogenously given budget on arms between itself and a rebel group operating in a target country, who must then divide its resources to meet both threats. Qiu shows that when arming the rebel group is cost-efficient, the target state can increase its share of the bargaining surplus by arming the rebel group.

3.2 Costly Peace

Though arming may be an efficient way to extract surplus in a one-shot game, it may not necessarily be efficient in the long run. If countries need to continuously invest in arms to extract a bargaining surplus, they may prefer to arm and then go to war in an attempt to eliminate their rival rather than repeatedly pay the cost of arming. This dynamic has been termed the “costly peace” dynamic, since it illustrates that peace can be “more inefficient than war" Coe (2011, p. 7).

For example, Israel’s military has long relied on reservists to provide the bulk of its military strength in times of war. In the lead up to the Six Day War, Israel mobilized large numbers of reservists to counteract an Egyptian mobilization and prepare for the conflict Churchill (1967, Ch. 3). Increasing numbers of troops remained mobilized for several weeks while diplomatic maneuvers and other preparations for war were being made. This mobilization put an immense strain on the economy, as the country’s workforce was drained to meet military needs. This dynamic contributed to Israel's decision to launch a war against Egypt on June 5th 1967.

To better understand the costly peace argument, consider the following simple example. Suppose that the bargaining game is now a stage game played repeatedly over an infinite number of discrete periods $t = 0, 1, 2, ..., \infty$. Both countries have a common discount factor $\delta \in (0, 1)$. We assume that country B cannot stockpile arms and so must repeatedly reinvest k to arm every period if it wishes to extract the bargaining surplus. Moreover, we will assume that if the countries go to war, then the stage game ends and the winner of the war will get to enjoy the share of the good $1-\theta$ that remains every period in perpetuity.

It turns out that this game has a unique equilibrium in which Country B always arms. To understand why, first observe that Country B can never achieve a utility larger than that which it could receive for going to war when it is armed. This is easy to see by noting that when it is armed, Country B will accept an offer if and only if they expect a stream of payments that satisfies

$$1 - x_t(1|h_t) + \delta V_t(x_t, h_t) \geq \frac{s(1-\theta)}{1-\delta}$$

Country A proposes so that the above holds with equality. On the other hand, if Country B doesn't arm, then the best payoff it can obtain is its payoff from going to war while weak or waiting a period and then arming (assuming Country A allows this) which we know must guarantee it a payoff of $\frac{s(1-\theta)}{1-\delta}$. Therefore, Country B will accept a demand when unarmed if and only if,

$$1 - x_t(0|h_t) + \delta V_t(x_t, h_t) \geq \max\left\{\frac{w(1-\theta)}{1-\delta}, \delta\left[\frac{s(1-\theta)}{1-\delta} - k\right]\right\}$$

Once again, the above must always hold with equality. It is therefore clear that Country B's utility will be larger when it arms.

The requirement that Country B always arm leads to a typical costly peace result, wherein a higher value of δ can lead to war. First, for sufficiently high δ,

$$\delta\left[\frac{s(1-\theta)}{1-\delta} - k\right] > \frac{w(1-\theta)}{1-\delta}$$

Second, to avoid going to war, Country B must receive a larger stream of payments that allows it to recoup its costs of arming if it is to avoid going to war. Suppose that Country A plays a Markov strategy, making the same offer every period that Country B arms (this is treated more completely in the appendix). Country B will, then only be willing to accept this offer if and only if

$$1 - x^* + \delta\frac{x^* - k}{1 - \delta} \geq \frac{(1 - \theta)s}{1 - \delta} \Longleftrightarrow 1 - x^* \geq (1 - \theta)s + \delta k$$

This implies that Country A must pay Country B more than its wartime payoff to avoid war, shrinking its share of the surplus. If Country A is sufficiently patient, it may prefer to go to war to seize this additional portion of the surplus. Formally, Country A will prefer to go to war if and only if

$$\frac{(1 - s)(1 - \theta)}{1 - \delta} > \frac{x^*}{1 - \delta} \Longleftrightarrow \delta \geq \frac{k}{\theta}$$

The following proposition summarizes these arguments.

Proposition 3 (Costly Peace). *There is a Markov Perfect equilibrium with the following features: Country B always arms. There exist a $\hat{\delta}$, such that for any $\delta > \hat{\delta}$, Country A makes an unacceptable offer and Country B goes to war. For any other δ, Country A always demands $x^* = 1 - (1 - \theta)s - \delta k$ and Country B always accepts.*

This description is a standard result in the literature on costly peace. Fearon (2018) studied a repeated game in which two countries simultaneously arm to improve their bargaining leverage over an issue. Separately, the two countries may go to war and fight one another for territory. He demonstrates, if countries are sufficiently patient then without fighting, countries could achieve cooperative equilibria with lower spending on arms than the one-shot game. It is the existence of the potential for a fight that can end the interaction that is necessary to generate war. Krainin and Wiseman (2016) present the strongest theoretical result in favor of the costly peace argument. They study a model where a network of countries may make peaceful transfers to one another or fight to disarm their rival permanently and extract their resources forever. They show that when countries are sufficiently patient, all states in the system will fight until only one state remains. Wiseman (2017) extends the arguments to firms in oligopolistic competition and demonstrates that sufficiently patient firms may prefer to initiate price wars that eliminate competition in the long-run than engage in collusive practices.

3.3 Secret Arming followed by Bargaining

We now turn to the situation where the militarization decision of B is not observed by A. This leads to a special case of the model in Meirowitz et al. (2022).

In any equilibrium of a game where B can arm, but the arming decision is not observed by A, A's conjecture regarding the strategy that B employs must be correct. Meirowitz and Sartori (2008) show that this hidden-action environment leads to the emergence of strategic uncertainty, where B employs a mixed, rather than pure, strategy in their arming decision. They show that this endogenous uncertainty is closely linked to the probability of war in bargaining. If there is a positive probability of arming, then there must also be a positive probability of war.

The logic for this result is straightforward. If B is arming in an equilibrium in which there is no chance of war, then a deviation to the strategy of not arming, but otherwise playing the same strategy would save the cost k, and not be detected by A. This is a profitable deviation. Consequently, the risk of fighting is a necessary form of discipline to support arming when militarization is a hidden action.

As in the previous model, only two possible offers are sequentially rational for A; the offer that makes an armed player B just indifferent between fighting and accepting and the offer that makes an unarmed player B just indifferent. Therefore, in this model, a risk of war requires that A sometimes make the offer that only the unarmed player B will accept and that B sometimes arms. We see then that war is only an equilibrium phenomenon if both countries employ mixed strategies; there must be uncertainty about B's militarization choice, and there must be uncertainty about A's behavior at the bargaining table.

To characterize such an equilibrium, we need two indifference conditions. First, suppose that B arms with probability q and A makes the offer giving B $(1-\theta)w$ with probability r and $(1-\theta)s$ with probability $1-r$. The second, conciliatory offer is accepted by B regardless of her arming choice, and the aggressive offer is accepted only if B has not armed. This leads to the indifference condition for A

$$1-(1-\theta)s = (1-q)(1-(1-\theta)w) + q(1-\theta)(1-s).$$

Additionally, in order for B to be indifferent, we have

$$(1-\theta)s - k = (1-r)(1-\theta)s + r(1-\theta)w.$$

This leads to the following result.

Proposition 4. *Suppose Assumption 1 is satisfied, then the equilibrium is unique. In equilibrium B arms with probability,*

$$q = \frac{(1-\theta)(s-w)}{\theta + (1-\theta)(s-w)},$$

A makes the offer $(1-(1-\theta)w, (1-\theta)w)$ with probability r, and the offer $(1-(1-\theta)s, (1-\theta)s)$ with probability $1-r$, with

$$r = \frac{k}{(1-\theta)(s-w)}.$$

An unarmed B accepts either offer, and an armed B accepts only the larger offer.

Importantly, this equilibrium assigns positive probability to the three different paths in the baseline deterrent result of Proposition 1.

4 Two-sided arming

Until now, we have focused on the case where only one country, B, can choose to arm or not. Now we explore the case where both countries can arm simultaneously and then bargain.[3] If the countries are mismatched, then the war payoffs are $s(1-\theta)$ for the stronger state and $(1-s)(1-\theta)$ for the weaker state. If they both have the same level of militarization, then the war payoffs are simply $\frac{1}{2}(1-\theta)$ for each state.

In this symmetric setting, it makes sense to have a symmetric bargaining protocol. We consider symmetric Nash demand bargaining (also sometimes called the $\frac{1}{2}$- double auction). In this bargaining game, each side makes a demand x_i. If the two demands sum to less than one, they are compatible with an efficient division of the prize and country i obtains $x_i + \frac{1-(x_A - x_B)}{2}$. If the demands exceed the size of the pie, then they fight and obtain their war payoffs.

In the spirit of Assumption 1, we make a similar assumption that arming is worthwhile. Specifically, if a country knows it is going to war, then it would rather arm against its opponent than to fight an evenly matched conflict.

Assumption 2. *For both states, the cost of arming satisfies*

$$(1-\theta)(s - \frac{1}{2}) \geq k$$

[3]Another literature on multilateral military spending looks at the general equilibrium effect of military spending on national wealth (Garfinkel 1990). This literature is related and ties state's arming decisions to tribute, but doesn't speak to bargaining or the onset of war.

	Arm	Not Arm
Arm	$1/2(1-\theta)+\theta/2-k$ $1/2(1-\theta)+\theta/2-k$	$s(1-\theta)+\theta/2-k$ $(1-s)(1-\theta)+\theta/2$
Not Arm	$(1-s)(1-\theta)+\theta/2$ $s(1-\theta)+\theta/2-k$	$1/2(1-\theta)+\theta/2$ $1/2(1-\theta)+\theta/2$

Table 1. Payoff to the bargaining game as a result of arming strategies

This assumption also is sufficient for a state to want to arm against an armed opponent and when it is true, implying Assumption 1.

4.1 Simultaneous public arming with bargaining.

We first consider the case where the arming decisions are observed before bargaining. In the bargaining game, there are many potential equilibrium outcomes. Given the symmetric nature of the game, we will focus on the equilibrium where the surplus is split evenly. This means that in a peaceful settlement, each country will demand and receive their war payoff plus half the surplus.

First, we show that there is a subgame-perfect equilibrium in which both countries arm.

Proposition 5. *Assume that A.2 holds. Then there exists a subgame-perfect equilibrium where both countries arm and there is a peaceful settlement at*

$$x_A^* = x_B^* = 1/2.$$

To see why this is the case, suppose both countries arm and in every bargaining subgame demand their war payoff (net of arming costs) plus $\theta/2$, given arming strategies. Payoffs to arming strategies in such an equilibrium are described as in Table 1. Payoffs from the split-the-surplus settlement, given that both arm, are

$$1/2(1-\theta)+\theta/2-k.$$

If a country deviates to not arming they get

$$(1-s)(1-\theta)+\theta/2,$$

which is not a profitable deviation if and only

$$1/2(1-\theta)+\theta/2-k \geq (1-s)(1-\theta)+\theta/2$$
$$s-\frac{1}{2} \geq \frac{k}{(1-\theta)}.$$

Last, observe that no country can demand more in any bargaining subgame without inducing war, which is strictly worse than the settlement.

Are there any other equilibrium arming strategies given the play in the bargaining subgames? By Assumption 2 we know that both countries would prefer to arm given the other has not, so there is no equilibrium where both don't arm. It is equally easy to show that it is worse to be unarmed versus an armed opponent than to be their equal. Therefore, in the equilibrium where Nash demand bargaining splits the surplus from avoiding war in every subgame, arming is a dominant strategy.

The equilibrium characterized in Proposition 5 is not unique, but it is perhaps natural, as it is fair and symmetric in this symmetric game. A continuum of other equilibria that are equally efficient can be found by focusing on uneven splits of the surplus. Note, however, that if the split of the surplus depends on arming choices, other equilibrium paths of play may be supportable.

The problem of two-sided arming and bargaining is studied in detail in Jackson and Morelli (2009). Their model adds some additional complications. One main contribution of their analysis shows that it can be in countries' interests to invest in levels of arms purely for deterrence. They argue that focusing on only two levels of militarization obscures this important fact, and they show that even if just three levels are possible, there can exist mixed strategy equilibria in which countries randomize over three levels of arms. Connecting with terms frequently used in the literature, they call these levels dove levels, hawk levels, and deterrent levels. Dove levels are low levels of arms which can be attacked and never deliberately choose to attack. Deterrent levels are those which are not attacked by some types of hawks, and are not the best responses to lower levels of arms. Hawkish arming levels are those which are never attacked and will sometimes attack.[4] Like our example, with binary levels of arms, the possibility of bargaining ensures that countries will not arm themselves to deterrent levels and doves will prefer to just pay off hawks. Interestingly, in their model, war occurs with positive probability even though military investments are public.

[4]Note that deterrent and hawk levels of arms need not be mutually exclusive, implying that there may exist equilibria in which countries arm themselves primarily to deter hawks, but are willing to fight certain levels of dove armaments.

	H	M	L
H	1/2 1/2	s $1-s$	S 1-S
M	$1-s$ s	1/2 1/2	w $1-w$
L	$1-S$ S	$1-w$ w	1/2 1/2

Table 2. Expected probability of winning a war with three arming levels

To illustrate this logic, consider a model like the one above, with minor changes. Suppose that instead of choosing to arm or not, countries can choose three different investments in arms, $\{L, M, H\}$. As before, assume that if fighting occurs, a fraction θ of the pie is lost. Furthermore, assume that the cost of high levels of arming, H, is $2k$ and the cost of deterrent arming, M, is k, and the cost of dovish arming, L, is 0. Finally, let Table 2 denote the fraction of the pie that remains from fighting, which that goes to A and B from each profile of arming levels (s_A, s_B) if war occurs.

For simplicity, we focus on the case where there is no bargaining. Rather than the Nash demand, game suppose now that each country simple decides to arm and then after the arming levels are observed, each country decides whether to fight or not. If either side selects to fight, then war occurs. To maintain symmetry, assume that the share of the pie going to each player is $\frac{1}{2}$ in a peaceful settlement.[5]

What is a symmetric completely mixed strategy equilibrium of this game? Let λ denote the probability of selecting L and μ the probability of selecting M, with the remaining probability going to H. In particular, we focus on a symmetric equilibrium in which each militarization level is played with strictly positive probability and war occurs at the pairs (H, L) and (L, H).

In such an equilibrium, the indifference conditions required in equilibrium are

$$(1-\lambda-\mu)(1-\theta)(1-S) + (\lambda+\mu)\frac{1}{2} = \frac{1}{2} - k$$

$$\lambda(1-\theta)S + (1-\lambda)\frac{1}{2} - 2k = \frac{1}{2} - k.$$

[5]Jackson and Morelli (2009) assume war decisions are sequential, this helps establish the equilibrium is unique. To maintain the symmetry of the game, we may assume that the war decisions are simultaneous and ignore profiles that involve weakly dominated play at the war stage.

The second equation simplifies to

$$\lambda^* = \frac{k}{S(1-\theta) - \frac{1}{2}}.$$

substituting this into the first condition, we obtain

$$\mu^* = (\frac{k}{S(1-\theta) - \frac{1}{2}})(\frac{(1-\theta)(1-S) - \frac{1}{2}}{\frac{1}{2} - (1-\theta)(1-S)}) - \frac{(1-\theta)(1-S) + k - \frac{1}{2}}{\frac{1}{2} - (1-\theta)(1-S)}.$$

We may now state a version of Proposition 1 in Jackson and Morelli for our model.

Proposition 6. *In the game with simultaneous arming with three levels and no bargaining, if $(1-\theta)S - 2k > \frac{1}{2}$ and $\frac{1}{2} - k > (1-\theta)w - k$ and $\frac{1}{2} - 2k > (1-\theta)s - 2k$, then in the unique equilibrium with weakly undominated strategies at the war stage, war occurs with positive probability (in pairs (H, L) and (L, H)) and each armament level is chosen with strictly positive probability. The equilibrium mixtures are given by λ^* and μ^* above.*

While a full proof can be constructed by applying the logic in Jackson and Morelli, here we flesh out the intuition for why a peaceful equilibrium cannot obtain. The parameter assumptions from the first condition imply that from a peaceful profile at (L, L) unilaterally selecting H and fighting is a profitable deviation. So we cannot have a peaceful equilibrium in which both states select L. But from a peaceful profile at (H, H), the last condition indicates that a unilateral deviation to M would save k and still deter fighting. From a peaceful profile at (M, M) the second condition indicates that a unilateral deviation to L would save k and still deter fighting. Thus, none of the symmetric profiles can support peace. Can an asymmetric profile support peace? Again, the answer is no because whoever is spending more could deviate to the arming choice of the other player and save at lease k and obtain deterrence. This means we must have mixing, and we must have a risk of conflict. This result is shown in the paper to carry over if we re-instate the possibility of bargaining to this three-level public arming model.

4.2 Simultaneous hidden arming with bargaining.

Next, we consider the case where the two countries can arm and then bargain, but the arming actions are unobserved. We will continue to assume that two evenly matched countries win the war with probability $1/2$ each and, when countries are unmatched in arms, the advantaged country wins with probability $s > 1/2$ and the disadvantaged country wins with the complementary probability $1 - s$.[6]

Here we maintain Assumption 1 and in fact we strengthen it to

Assumption 3. *The payoff from fighting an imbalanced war while armed is sufficiently high, namely*

$$s(1-\theta) \geq \frac{1}{2}$$

After militarization decisions have been made, countries attempt to negotiate a peaceful agreement and avoid a destructive war. Like in the previous section, we represent negotiations as a Nash demand game. Players simultaneously make demands x_A and x_B both in $[0,1]$. If $x_A + x_B \leq 1$, then each player i receives a split of the pie equal to its demand x_i plus half the surplus $1 - x_A - x_B$. If the demands x_A and x_B are incompatible, i.e., if $x_A + x_B > 1$, then the outcome is war.

To make the exposition parsimonious, we will focus on symmetric arming strategies and introduce the parameter $\gamma = (s(1-\theta) - 1/2)/(1/2 - (1-\theta)/2)$, which subsumes the two parameters θ and s. The numerator of γ is the gain of an armed country from waging war against an unarmed country instead of accepting the equal split. The denominator is the loss from waging war against an armed country rather than accepting the equal split. So, γ represents the ratio of benefits over cost of war for an armed country.

We follow Meirowitz et al. (2019) and select equilibria in which bargaining behavior maximizes the probability of peace (among equilibrium strategies) given the equilibrium probability of arming. We begin our analysis by solving for mutual best responses at the negotiations stage, after militarization decisions have been made. That is, we solve the Nash demand game holding the militarization probability q fixed.

As a function of q, Proposition 7 reports the equilibrium of the Nash demand game that maximizes the probability $V(q)$ of peaceful resolution of the dispute as in Meirowitz et al. (2019).

Proposition 7. *As a function of the arming probability, q, the equilibrium of the Nash demand game that maximizes the peace probability is as follows: For $q \geq \gamma/(\gamma+1)$, the countries always achieve peace by playing $x_A = x_B = 1/2$. For $\gamma/(\gamma+1) > q \geq \gamma/(\gamma+2)$, peace is achieved unless both countries arm; armed countries demand $x_H \in [p(1-\theta), 1-(1-p)(1-\theta)]$ and unarmed countries demand $x_L = 1 - x_H$. For $q < \gamma/(\gamma+2)$, peace is achieved only if both countries are unarmed, unarmed countries demand $x_L = 1/2$, whereas armed countries trigger war by demanding $x_H > 1/2$.*

[6] This section is based on one variant of the model in Meirowitz et al. (2019).

Meirowitz et al. (2019) show that equilibrium bargaining varies as countries' expectations about arming change. The relevant cut-off values for q are $\gamma/(\gamma+1)$ and $\gamma/(\gamma+2)$. For $q \geq \gamma/(\gamma+2)$ both countries are cautious because the probability their opponent armed is high. Since war with an armed opponent gives a payoff of $(1-\theta)/2$ both countries are willing to choose a settlement that splits the prize and the outcome is a peaceful settlement. On the other hand, when $\gamma/(\gamma+2) \leq q \leq \gamma/(\gamma+1)$ armed countries are willing to risk war, making higher demands, while unarmed countries, fearing war, make low demands that will placate an armed adversary. If $q < \gamma/(\gamma+2)$, however, even unarmed countries make higher demands that trigger war with armed opponents, but result in peace if their opponent is unarmed.

Given these results, and their implications for the consequences of arming strategies, a cost k, Proposition 8 describes the equilibrium strategies that maximize the countries' welfare.

To focus on the clearest case, Meirowitz et al. (2019) make the following assumption.

Assumption 4. $k \geq \underline{k} \equiv \theta\gamma/2 \cdot \gamma/\left(\gamma+1\right)$ *and* $k \leq \overline{k} \equiv \theta\gamma/2 \cdot \left(\gamma+1\right)/\left(\gamma+2\right).$

Proposition 8. *When the cost of arming, $k \in [\underline{k}, \overline{k}]$, the equilibrium of the militarization and negotiation game that maximizes the players' welfare W is such that each player militarizes with probability $q(k) = \gamma - 2k/\theta \in \left[\gamma/\left(\gamma+2\right), \gamma/\left(\gamma+1\right)\right]$, strong players demand $x_H = s(1-\theta)$, weak players demand $x_L = 1 - s(1-\theta)$, and war breaks out if and only if both players are strong.*

We do not reproduce the proof from Meirowitz et al. (2019) here. For completeness, however, it is worth presenting an overview of the logic behind this result.

The argument can be taken in two steps. First, when the arming cost k lies between $\underline{k}$ and $\overline{k}$, there is an equilibrium of the game in which countries arm with probability $q^*\left(k\right)$, armed countries make a high demand $x_H = s(1-\theta)$ when bargaining, unarmed countries demand $x_L = 1 - s(1-\theta)$, and war breaks out if and only if both countries arm. So, when the militarization cost k is neither too small, nor too large, countries are willing to randomize their arming decision at 'intermediate' values. This is the range in the previous result. At the negotiation stage, the strategies of this equilibrium are consistent with the equilibrium selected by Proposition 7. Hence, this equilibrium maximizes the peace probability at the negotiation stage.

The second step involves showing that there cannot exist any other equilibrium with a smaller militarization probability. Notice that the equilibrium of Proposition 8 yields the highest possible payoffs to unarmed countries, the lowest payoffs to armed countries and, therefore, the lowest *ex-ante* incentives to militarize. Because the equilibrium of Proposition 8 minimizes the arming probability q at the militarization stage, and maximizes the peace probability in the ensuing negotiation stage, it immediately follows that it maximizes welfare in the whole game.

Meirowitz et al. (2019) analyze this game as part of a large exercise to understand whether mediated cheap talk can outperform a variant of this game with cheap talk. The answer is yes, and perhaps most interestingly, they show that a mediator that is mandated only to minimize the odds of war for a current conflict (that is she takes q as exogenous) will act in ways that also minimize the incentives to arm. In other words, solving the short run problem that takes arming choices as given happens to solve the long-run problem in which militarization is endogenous.

We see here that while arming can play a role in deterring conflict, unobservable arming and two-sided arming with several levels can lead to war. The first fact is consistent with our existing understanding of the effects of uncertainty on the risk of conflict. The second, that multiple levels of arming can lead to war, is counterintuitive and suggests that policies, like open skies, that allow arming to be observed, may not be sufficient to prevent war.

5 Signaling strength and costs and types with bargaining

Several scholars have developed accounts of militarization as a signal of pre-existing uncertainty about a country's willingness to fight to change the status quo distribution of rents, whether from policy or territorial control. The common components in these models are that one country can arm and some component of the arming country's payoff is private information. After observing the decision whether or how much to arm, the rival country can draw inferences about the rival's type, update its beliefs, and then decide whether to stand firm and fight or concede to a demand. [7]

Slantchev (2005) studies a signaling model in which the signal is made by arming and the costs of mobilization are sunk. The article has several results, but one key finding relates to how signaling through arming affects the probability of war.

To see how this works, we consider a variant of the baseline model with private information. Suppose state B has two potential values for an indivisible good, $\{\bar{V}_B, \underline{V}_B\}$,

[7] Note that states need not have their investment in arming succeed in improving their probability of victory to signal successfully. Early models of signaling in conflict demonstrated that simply burning money could be used to signal a state's willingness to fight over a good or issue (Fearon 1997). However, later scholars pointed out that military mobilization is a form of sunk cost investment that states make in diplomatic crises and that pure money burning is rare (Slantchev 2005).

each with equal probability. Also assume that A receives utility of U_A even if they forfeit the good, and prefers capitulating to fighting an armed opponent. That is, $U_A > (1-s)(1-\theta)$.

At the beginning of the game, nature determines B's valuation of the good. Next, B chooses to arm or not. Arming changes A's probability of winning a conflict for the prize to $1-s$ from $1-w$. Upon seeing the arming choice of B, A decides whether to capitulate and give the good to B or call B's bluff and force B to choose to fight or back down. At that point, B chooses to fight or back down. Backing down yields a payoff of U_B to B, net the sunk arming costs.

Here arming serves two purposes, it signals the intensity of B's interest and affects the payoff to war directly. We extend our simplified model by imposing the following assumptions, which arise endogenously in Slantchev's (2005) original model.

Starting at the final decision to fight or back down, suppose that when B is the low-value type, it is indifferent between fighting and backing down when they arm, and strictly prefers backing down when unarmed. When B is the high-value type, they prefer to fight regardless of how they arm, but arming produces a higher probability of success. Formally, we assume

$$\underline{V}_B s(1-\theta) = U_B,$$
$$\underline{V}_B w(1-\theta) < U_B,$$
$$\bar{V}_B s(1-\theta) > \bar{V}_B w(1-\theta) > U_B.$$

The first assumption is knife-edged, but Slantchev shows how the analogue to this condition comes about more naturally in the richer model. Finally, like Assumption 1, assume $\bar{V}_B(1-\theta)(s-w) > k$.

We can now construct a signaling equilibrium.

Proposition 9. *Given the assumptions of the signaling game, there exists a perfect Bayesian equilibrium where the high-value type of B arms with probability one, the low-value type mixes between arming and not, state A mixes between calling B's bluff, high-value types fight with probability one and armed low-value types fight with positive probability.*

To see that this is an equilibrium, start by observing that the high-value type of B trivially prefers to fight and since the armed low-value type is indifferent, it can mix. Therefore, A is willing to mix and call B's bluff with positive probability when, given interim belief $\mu = Pr(t_B = \bar{V}_B | a)$ and mixture by the armed low-value of B when its bluff is called is

$$U_A = \mu(1-s)(1-\theta) + (1-\mu)(\phi(1-s)(1-\theta) + (1-\phi)(1)),$$
$$\phi^* = \frac{\mu(1-(1-s)(1-\theta)) + (1-U_A)}{(1-\mu)(1-(1-s)(1-\theta))}.$$

$\phi^* < 1$ as long as $\mu < \frac{1-U_A}{(1-(1-s)(1-\theta))}$, where μ is endogenous to B's arming action.

Next, let π^* be the probability that A calls B's bluff after arming. Given ϕ^*, π^* is chosen to make the low-type of states B is indifferent between arming and not, we need

$$(\pi)(Pr(war)0 + (1-Pr(war))0) + (1-\pi)\underline{V}_B - k = U_B,$$
$$(1-\pi)\underline{V}_B - k = U_B$$
$$\pi^* = \frac{\underline{V}_B - k - U_B}{\underline{V}_B} < 1.$$

Finally, since the low-value type of B's expected payoff is zero regardless of their action, B can choose any mixture σ that generates the required belief μ, and by Bayes rule, $\mu = \frac{1}{1+\sigma}$, satisfying the condition.

Notice that if arming had no effect on war payoffs, i.e., if $s = w$, then this equilibrium would break down. The low-value type would not fight when armed, and as a result, these strategies would unravel. War would then only be possible in any equilibrium with high-value types, making signaling through increasing a state's military capabilities a pernicious problem.

5.1 Robustness of the Signaling Result

Several scholars have demonstrated that the existence of signaling equilibria and the comparative statics arising from signaling models are sensitive to assumptions about the source of uncertainty (Arena 2013; Carroll and Pond 2021; Reich 2023), game form (Wolton 2023), and assumptions regarding signaling costs (Reich 2023). For example, Reich (2023) demonstrates that if there is uncertainty regarding a country's initial strength, then weaker states can actually arm more. This is because stronger states are more tolerant of war than their weaker counterparts. Consequently, arming is indicative that the state is a low type.

To see how this works, consider a similar signaling game as before, but in which Country B has private information regarding its probability of winning a war. In this simplified version of Reich (2023), assume that Country B can now be one of two possible types $\omega \in \{L, H\}$, each with equal probability, where the low type's probability of winning a war is given by $\underline{s}$ if it armed and $\underline{w}$ if it is not. Similarly, the high type's probability of winning is given by $\bar{s}$ if it is armed and $\underline{w}$ if it is not. To have these types reflect uncertainty about a country's initial strength, we will have $\underline{w} < \bar{w}$ and $\underline{s} < \bar{s}$. For simplicity's sake, we will assume that arming increases each Country's probability of winning by the same amount, regardless of its type. Moreover, we will assume that Country A would always prefer to concede to an armed type of Country B, would strictly prefer to fight an unarmed low type but would strictly prefer to concede in response to an unarmed high type. Let U_A denote Country A's utility from conceding. Formally, these assumptions can be written as

$$\begin{aligned} \bar{s} - \bar{w} &= \underline{s} - \underline{w} \\ U_A &> (1 - \underline{w})(1 - \theta) \\ U_A &< (1 - \underline{s})(1 - \theta) \end{aligned}$$

As before, Assumption 1 will still hold and ensure that Country B will always find it worthwhile to arm if it expects a fight.

Once again, there exists a signaling equilibrium. However, this time it is the high types who do not arm.

Proposition 10. *Given the assumptions of the signaling game, there exists a Perfect Bayesian equilibrium where the high type of B never arms, and the low-value type mixed between arming and not, Country A always concedes in response to arming, Country A mixes between calling B's bluff when it is unarmed, and Country B always fights unarmed if Country A does not concede.*

To see how to construct such an equilibrium, let π denote the probability that Country A concedes if Country B does not arm. The low type of Country B will be indifferent between arming whenever

$$\pi + (1 - \pi)(\underline{w})(1 - \theta) = 1 - k$$

such that

$$\pi(\underline{w}) = \frac{1 - \underline{w}(1 - \theta) - k}{1 - \underline{w}(1 - \theta).}$$

Similarly, the high type of Country B will be indifferent between fighting and not when

$$\pi(\bar{w}) = \frac{1 - \bar{w}(1-\theta) - k}{1 - \bar{w}(1-\theta)}$$

It is straightforward to see that $\pi(\bar{w}) < \pi(\underline{w})$, implying that the probability of concession that makes the high type of Country B indifferent between arming and not is strictly smaller than the probability that makes the low type of Country B indifferent. This reflects the high type's increased tolerance for fighting.

All that is needed to complete the construction of the equilibrium is to find the probability with which a low type of Country B can arm that leaves Country A indifferent between calling B's bluff or not in response to no arming. Formally, this requires that

$$U_A = \mu(1-\bar{w})(1-\theta) + (1-\mu)((1-\underline{w})(1-\theta))$$

so that Country A's belief that Country B is the low types is

$$\mu = \frac{(1\underline{w})(1-\theta) - U_A}{\bar{w} - \underline{w}}(1-\theta)$$

the above value of μ is strictly greater than 0 given our assumption that $(1\underline{w})(1-\theta) > U_A$. Moreover, it is simple to check that $\mu < 1$ under the assumption that $(1-\bar{w})(1-\theta) < U_A$. Therefore, the low type of Country B simply arms at the probability required to achieve this belief.

There are several models exploring a different set of environments that produce additional interesting substantive results. For example, Reich (2022) argues that states may signal strength by handicapping, under-utilizing or under-deploying their military strength, to demonstrate their tolerance for fighting while unarmed and in doing so can communicate strength. Slantchev (2010), demonstrates that when a state's rival can mobilize in response to demands, states may choose to disguise their type to lull their into complacency and ambush them.

6 Preventive attack of a state that can arm

We close, by moving from models that focus on strategic militarization to models that focus on actions that a state may take in response to militarization by an opponent.

The most well studied of these actions is preventive attack. Scholars of international relations have been increasingly interested in the following type of narrative: On March

20, 2003, the US and a number of its allies invaded Iraq and toppled its dictator Saddam Hussein. Their stated justification for the war was Hussein's ostensible pursuit of weapons of mass destruction (WMDs). Accepting this justification at face value, the attack on Iraq can clearly be understood as a case of preventative war - the US attacked believing that it possessed a clandestine weapons program that had the potential to rapidly alter the balance of power. Moreover, if the US attacked immediately, then it could destroy Iraq's weapons program and preserve the balance of power in its favor. However, after the US completed its occupation of Iraq, it quickly became apparent that contrary to the US's beliefs, Iraq did not have a secret weapons program.

Inspired by these events, several scholars have studies models of arming in which one country can secretly decide whether to arm, the decision to arm produces a noisy signal, and there is a delay between the investment in arming, and its taking effect. This affords the rival an opportunity to launch a preventative war and prevent the arming from taking place. In such models, preventative war results from a commitment problem- if arming shifts power by enough, or for long enough, then the arming country may not be able to compensate its rival for the anticipated shift in strength (Powell 2006). However, even if this condition does not hold, secret delayed arming can still result in war but the mechanism driving it will not be a commitment problem.

To provide traction on this intuition, consider the following canonical two period-model from Debs and Monteiro (2014). In period one, country B makes an unobserved binary arming decision. It may arm at cost k. Though country A does not observe country B's decision, it receives a noisy signal $\omega \in \{0,1\}$. Let $Pr(\omega = 0|m_b = 0) = 1$ and $Pr(\omega = 1|m_b = 1) = p_k$ with $p_k \in (\frac{1}{2}, 1)$. After observing the signal ω, country A must decide whether it will offer a peaceful division $(x, 1-x)$ or go to war. If country B does not accept the peaceful division, then war occurs. If no war occurs and country B has chosen $m_B = 1$, then its investment becomes common knowledge and country B's strength increases from w to s in the second period, in which country A makes a take-it-or-leave-it offer. However, if war occurred in period 1, then country B remains unarmed regardless of its investment decision and both countries receive their wartime payoffs in both periods.

This game has a unique equilibrium which can be arrived at via backwards induction. First, if a war does not occur in the first period, then country A will offer country B its wartime payoff in the second period. Anticipating a larger offer will need to be made to B if it is armed in the second period, country A will be willing to accept an offer smaller than its payoff for fighting in the first period

$$1 - x_a = w(1-\theta) - \delta(s-w)(1-\theta).$$

This offer is the minimum offer that keeps country B indifferent between fighting unarmed in both periods and accepting $1 - x_a$ in the first period, and $s(1-\theta)$ in the second. Note that this implies that country B can only benefit from arming if it manages to surprise country A when it does so. Indeed, if country A learns that country B has armed, it can adjust its first-period offer down such that the stream of offers is equivalent to fighting a weak type of country B for two periods. As a result, if country B's decision to arm is revealed then country A offers $1 - x_a$ in period 1, $s(1-\theta)$ in period 2, country B accepts both offers, war does not occur, and country B loses k because of their decision to arm.

Consequently, a necessary condition for country B to arm is that its decision to do has a sufficiently high probability of remaining a secret. To keep matters interesting, we will make the following Assumption

Assumption 5. *The rising state's probability of being discovered following its arming decision is low enough to justify the expense*

$$(1-p_k)(s-w)(1-\theta) > k$$

This condition closely resembles Assumption 1, but is more restrictive. If this condition does not hold, then country B's decision to arm is too likely to be discovered, and it can expect a stream of payoffs equal to its payoff of fighting while weak, without recouping k.

If Assumption 2 holds, then any equilibrium must be in mixed strategies. To see why, note that if country B always armed, then country A would simply offer $1 - x_a$ in the first period, in which case country B has a profitable deviation to not arm. By contrast, if country B never arms, then country A would offer $1 - x = w(1-\theta)$ in which case, country B has a strictly profitable deviation to arming. Therefore, country B must mix whether it arms or not, and given a signal that $\omega = 0$, country A must mix whether it makes the low offer $1 - x_a$ that only an arming type would accept or a high offer $1 - x = w(1-\theta)$ that country B will always accept. If country B has not armed, and receives the high offer, they will prefer to reject it and go to war since they do not expect a surge in strength in period 2. Therefore, the mechanism by which war occurs in this example is a simple risk-reward trade-off (Slantchev and Tarar 2011).

It is straightforward to characterize the mixed strategy equilibrium. Let q denote the probability with which country A makes the low offer $1 - x_A$, and with reciprocal probability makes the offer $1 - x = w(1-\theta)$. In this case, country B's expected utility for arming is given by

$$p_k[1 - x_a + \delta s(1-\theta)] + (1-p_k)[q(1 - x_a + \delta s(1-\theta)) + (1-q)(w(1-\theta) + s(1-\theta)] - k$$

and their expected utility for not arming is always simply

$$w(1-\theta) + \delta w(1-\theta)$$

. Setting the two equal to each other, we find that country A makes the low offer with probability,

$$q^* = \frac{\delta(1-p_k)(s-w)(1-\theta) - k}{\delta(1-p_k)(s-w)(1-\theta)}$$

Let r denote the probability with which country B arms. country A's expected utility for making the low offer, conditional on a signal $\omega = 0$ is given by

$$\frac{r(1-p_k)}{(1-r) + r(1-p_k)}[x_A + \delta s(1-\theta)] + \frac{1-r}{(1-r) + r(1-p_k)}[(1-w)(1-\theta) + \delta((1-w)(1-\theta))]$$

and their expected utility for making the high offer conditional on a signal of $\omega = 0$

$$1 - w(1-\theta) + \delta \frac{r(1-p_k)}{(1-r) + r(1-p_k)}[s(1-\theta] + \delta \frac{1-r}{(1-r) + r(1-p_k)}(1 - w(1-\theta))$$

Setting the two equal to each other, we find that country B arms with probability

$$r^* = \frac{\theta + \delta\theta}{\delta(1-p_k)(s-w)(1-\theta) + \theta + \delta\theta}$$

We can therefore summarize the result in the following proposition.

Proposition 11. *If Assumption 5 holds, then there is a unique equilibrium to the two-period hidden arming game in which country B arms with probability r^*. If $\omega = 1$, then country A demands x_A in period 1 and $1 - s(1-\theta$ in period 2 and country B accepts both offers. Otherwise, country A demands x_A with probability q^* and $1 - w(1-\theta)$ with probability $1 - q^*$ in period 1. In response, country B always accepts $1 - (w1 - \theta)$ but rejects $1 - x_A$ if it has not armed. If the game proceeds to a second period, then country B demands $1 - s(1-\theta)$ if country B is armed and $1 - w(1-\theta)$ otherwise.*

6.1 Infinite-Horizon Model

Movement from the two-period model to longer horizon models demonstrates an interesting intuition: if arming shifts power by a sufficiently large amount for long enough, then country A will go to war to prevent it. To see this, consider the following longer horizon model also based on Debs and Montero (2014). In the game, time is indexed by $t = 0, 1, 2, ..., \infty$ and both countries discount the future at rate δ. In each period, countries play the following stage game. First, country B gets to choose whether to arm or not at cost k. country A does not observe country B's decision. However, if country B decides to arm, country A receives a noisy signal $\omega \in \{0, 1\}$ where $\omega = 1$ indicates that country B has chosen to arm and $\omega = 0$ indicates that it has not. Note that it is possible to retain a two period model and capture this intuition by changing the magnitudes of some payoffs, but in the interest of providing a broader perspective on modeling strategies in the literature we chose to present an infinite horizon extension.

For simplicity, assume that if $Pr(\omega = 0|m_b = 0) = 1$ and that $Pr(\omega = 1|m_b = 1) = p_k$ with $p_k \in (\frac{1}{2}, 1)$. This implies that a signal $\omega = 1$ implies that country B has chosen to arm, whereas a signal of $\omega = 0$ need not imply that country B has not armed. After observing the signal ω, country A must decide whether it will offer a peaceful division $(x, 1 - x)$ or go to war to determine the division for that period. If country B does not accept the peaceful division, then war occurs. To keep matters simple assume that if a war occurred, then both country's obtain their wartime payoff forever.

Finally, assume that country A would be willing to launch a preventative war if it were known that country B were arming and was willing to accept a demand of $x = 1$ in the current period and $1 - s(1 - \theta)$ in every period thereafter, i.e.,

$$\frac{(1-w)(1-\theta)}{1-\delta} > 1 + \delta\frac{1-s(1-\theta)}{1-\delta}$$

This can be simplified in the form of the following assumption

Assumption 6. *country A is willing to launch a preventative war*

$$(\delta s - w)(1 - \theta) > \theta$$

Note, that if this assumption does not hold, then war can still result via a risk-reward trade-off mechanism as it did in the two-period example. Additionally, we will assume that country B's probability of getting caught while arming is sufficiently low to justify trying to arm,

$$p_k\left(\frac{w(1-\theta)}{1-\delta}\right)+(1-p_k)\left(\frac{s(1-\theta)}{1-\delta}\right)-k>\frac{w(1-\theta)}{1-\delta}$$

which can be simplified in the following assumption

Assumption 7. *country B is willing to risk arming if*

$$(1-p_k)(s-w)(1-\theta)>k(1-\theta).$$

This Assumption is more easily satisfied than Assumption 5 since the cost of arming can be recouped over a larger number of periods. If this assumption does not hold, then the threat of country B's decision to arm being discovered and preventative war effectively deter country B from arming.

However, if both assumptions hold, then the game has an equilibrium in mixed strategies. Let q denote the probability that country A attacks following a signal of $\omega = 0$ in any given period. In this case, country B's expected utility from arming is given by

$$[p_k+(1-p_k)q]\frac{w(1-\theta)}{1-\delta}+(1-p_k)(1-q)\left[w(1-\theta)+\delta\frac{s(1-\theta)}{1-\delta}\right]-k$$

and their expected utility for not arming is given by

$$\frac{w(1-\theta)}{1-\delta}.$$

Setting the two equal, we find that country B will be indifferent between fighting and not when country A responds to a signal ω by going to war with probability

$$q^*=\frac{\frac{(s-w)(1-\theta)}{1-\delta}\delta(1-p_k)-k}{\frac{(s-w)(1-\theta)}{1-\delta}\delta(1-p_k)}.$$

Similarly, country A's expected utility from attacking following a signal of $\omega = 0$ is given by

$$\frac{(1-w)(1-\theta)}{1-\delta}$$

and their expected utility from not attacking is given by

$$V(0) = 1 - w(1-\theta) + \delta \frac{r(1-p_k)}{r(1-p_k)+(1-r)} \frac{1-s(1-\theta)}{1-\delta}$$
$$+\delta \frac{1-r}{r(1-p_k)+(1-r)} \left[rp_k \frac{(1-w)(1-\theta)}{1-\delta} + (1-rp_k)V(0) \right]$$

simplifying the expression for $V(0)$ and combining setting the two equations equal to each other, we find that country A will be indifferent between attacking and not when country B arms with probability

$$r^* = \frac{\theta}{(1-p_k)(s-w)(1-\theta)+\theta p_k}.$$

We can summarize the result in the following proposition.

Proposition 12. *If Assumptions 6 and 7 hold, then there exists an equilibrium to the infinite-horizon repeated arming game in which country B arms with probability r^*. If $\omega = 1$ in any period, then country A goes to war. Otherwise, country A goes to war with probability q^* and demands $1 - w(1-\theta)$ with probability $1 - q^*$ which country B accepts.*

There exist many ways in which scholars have sought to add additional features to the above model. For example, Debs and Monteiro (2014) consider an environment in which war is not game ending but instead prevents country B from arming for N periods, and find that as N increases the probability of preventative wars, including mistakenly launched ones, goes up. Others have sought additional mechanisms with which to explain the Iraq war, particularly why Iraq did not reveal that it had not armed to inspectors. Coe and Vaynman (2020) argue that disclosing information about arming decisions may weaken country B and cause them to withhold information. Baliga and Sjöström (2008) study a model in which country A can attack after arming has been realized and country B has been afforded the opportunity to reveal its arms to explain why Iraq benefited from strategic ambiguity. The key innovation here is to have the arming country potentially be a crazy type, who is more likely to arm and who gives its rival a worse payoff if it is armed. In this case, revealing that one is armed might incentivize an attack because it makes the rival believe that the armed country is a crazy type. On the other hand, revealing that one is unarmed can incentivize an attack.

Some authors have studied a similar problem in which a country can take costly measures to effectively disarm their rivals. Observing that countries often take measures short of preventative wars to prevent their rival's from arming, Schram (2021) studies a model in which countries can decide how much to "hassle" their rival to mitigate an exogenous power increase in the next period. He shows that when such measures are effective and war is costly, a country may repeatedly choose to hassle a rival rather than launch a preventative war. Coe (2018) argues that the US was containing Iraq in this way until a decrease in the efficacy of containment led it to go to war. In a companion paper, Schram (2022) studies a model in which a country can endogenously decide whether and how much to arm and its rival can respond by hassling or launching a preventative war. Surprisingly, he shows that the ability to take actions short of war can actually make the hassling country worse off, since it makes the threat of war less credible and embolden its rival to arm.

7 Conclusion

This review has explored how models of strategic interaction have been used to study militarization decisions and their impact on negotiations and the likelihood of war. Through the development of a baseline model and various extensions, we have highlighted key forces that shape militarization incentives and outcomes.

A core insight is that the ability of states to commit to alternative resource divisions limits inefficient fighting, but opportunities for investment in strength that change war payoffs continue to impact negotiated agreements. When militarization is observable, it can serve as a deterrent. However, when it is unobservable, endogenous uncertainty emerges, increasing the risk of war. Whether arming is framed as a pre-commitment versus a response also matters. If considered a commitment, a state may stand firmer in subsequent bargaining. As a response, sunk costs limit what a state can credibly demand.

Signaling motivations feature prominently in some models. Militarization may signal resolve or strength and deter aggression. However, it can also provoke arms races. Allowing for more than two levels of arms demonstrates possibilities for investing purely to deter the strongest types of opponents. Preemptive attack models underscore risks that incentivize preventative war when power shifts are substantial.

Across models, the probability of warfighting and the extent of wasteful arming provide natural welfare metrics. Hidden arming and signaling tend to increase conflict odds versus observable decisions. Uncertainty over the costs and benefits of arming for a potential aggressor results in higher expenditures. Allowing states to endogenously choose continuous investment levels mitigates distortions relative to a binary choice.

While formal modeling provides insight into core tradeoffs, most work recognizes limitations of sparse representations of bargaining, unitary actors, and omitted factors.

Reality involves more complex negotiations, domestic constraints, and other factors influencing choices. Models help clarify causes of inefficient fighting, but cannot definitively explain specific historic cases. Limitations motivate incorporating empirical evidence and moving to richer settings, as done throughout the literature.

8 Appendix

8.1 Markov Strategies in the Costly Peace

Here we demonstrate that any SPE resulting in peace requires Country A to make a unique sequence of identical offers. To see why, recall that Country B will accept a demand if and only if

$$\sum_{\tau=t}^{\infty} \delta^{\tau-t} x_\tau(1|h_\tau) \geq \max\left\{\frac{w(1-\theta)}{1-\delta}, \delta\left[\frac{s(1-\theta)}{1-\delta} - k\right]\right\}$$

Let x^* be the offer for which the above holds with equality when it is offered repeatedly in every period. Country A will only make offers such that the sequence holds with equality. In principle, however, it is possible for the equality to hold if when offers fluctuate; if Country A makes a demand $x_t(1|h_t) < x^*$ in one period, it could maintain the equality B by making larger demands $x_t(1|h_\tau) > x^*$ in a future period (or possibly multiple periods). However, such a strategy does not survive the one-shot deviation principle. To prove this, it is sufficient to observe that $1 - x^*$ is the minimum offer that Country B would accept to delay war for a single period. Therefore, any alternate strategy must feature at least one period in which Country A makes a demand x' that is smaller than x^* and must contain a period in which it could achieve a higher payoff by increasing its demand to x^* (and making that offer forever thereafter) as Country B must accept the offer.

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