Conservation for Sale: International Bargaining over Payment for Ecosystem Services [*]

Kristopher W. Ramsay[†] and Noam Reich[‡]

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Abstract

Recent years have seen a proliferation of international conservation agreements. Under the UN's program for Reducing Emissions from Deforestation and Forest Degradation (REDD+) developing countries can receive payments from interested parties in exchange for conserving their forests. We study such programs and ask when countries can reach agreements to conserve natural resources. Specifically, we analyze a formal model where a developing country controls a limited non-renewable resource that they prefer to consume, while another country prefers that it be preserved. We present four main results. First, successful conservation is only possible once the forest is sufficiently small. Second, there exists a trade-off between conservation and the share of bargaining surplus that goes to the developing country. Third, by allowing leakage, developing countries can increase their rents from agreement, but simultaneously undermine conservation. Finally, conservation is maximized and leakage averted when developing countries dictate the terms of agreements.

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[†]Princeton University. Email: `kramsay@princeton.edu`.

[‡]NYU Abu Dhabi. Email: noam.reich@nyu.edu

1 Introduction

In 2020, Brazil proposed that the developed world make it an annual payment of 10 billion USD to cover the cost of addressing the numerous challenges related to the conservation of the Amazon biome (Harris 2020). Similarly, but with a much smaller price-tag, in 2007 Ecuador asked wealthy countries to pay them 350 million USD a year, for 10 years, in exchange for leaving an estimated one billion barrels of oil under the ground in the pristine Yasuni rainforest (NY Times 2007). Neither country was paid.

These are but two example of a recurring conflict. The developed world wants developing countries to forego the use of natural resources that have economic value. In return, the resource owners want compensation for forgoing the consumption of those resources. A common manifestation of this dilemma is the problem of forest management and conservation. International actors value the forest's ability to sequester carbon and support biodiversity. The home countries, who may also value these things, see the economic value of timber and alternative land use that can improve the lives of their citizens. On the one hand, deforestation and forest degradation is a major cause of carbon dioxide equivalent emissions (C02e), accounting for 12% of global anthropogenic CO2e emissions by recent estimates (Van Der Werf et al. 2009). At the same time, Mullan et al. (2018) show that converting rainforest to agricultural land leads to significant increases in household income and the accumulation of wealth, which facilitates access to education and medicine, a reduction in poverty, and generally higher standards of living.

In response to these competing goals, the international community negotiated a framework designed to facilitate conservation agreements. Under the United Nation's program for Reducing Emissions from Deforestation and Degradation (REDD+), adopted in 2013, developed countries can pay developing ones not to consume their forests. REDD+ consists of a set of guidelines for agreements that play an important role in reducing transaction costs. However, it leaves the amount conservationists should pay forest owners for their conservation efforts unspecified. As the examples cited above illustrate, this can prevent agreements from being reached.

At its core, the fundamental problem is that someone who does not own a particular resource wants to prevent its consumption. Unlike typical international environmental issues, which focus on public good provision or the tragedy of the commons (Barrett 2005), international cooperation related to conservation is best thought of as a Coasian problem in which two parties bargain to address the externalities that one's actions impose on the other (Coase 1960; Muradian et al. 2010). In this case, the issue is: What kinds of mutually beneficial agreements are possible? What are the fundamental strategic constraints on payment for conservation programs?[1]

To answer these questions, we develop a theory of bargaining over conservation. We consider the case where one country controls a limited non-renewable resource that they prefer to consume, while another country prefers that it be preserved. The consumption process is dynamic, in that the owner can consume one unit of resource at a time, and there is limited commitment on the owner's part. In particular, the owner cannot sell the resource to the conservationist, and they cannot promise to conserve, or even consume, the resource when it is not in their interest to do so. This strategic interaction is naturally described by a dynamic game where proposed per period payments are made, followed by decisions from the resource holding country to consume or not. This strategic setting could apply to a number of different goods including forests, endangered species, or peat deposits. Since the strategic incentives are the same across these resources, it is easiest to write, and think, in terms of a single resource, forests. Like Harstad (2016), hereafter we will treat the resource as a tropical forest, refer to a unit of the resource as a tree, and to the unconsumed stockpile of the resource as a forest.

Our analysis focuses on answering three questions: How is the bargaining surplus divided between the forest's owner and the conservationist outsider? What is the greatest amount of conservation can hope for when actors bargain over a good that has both economic and environmental value? How does the distribution of benefits and the amount of forest conserved vary as a result of the framework within which agreements are reached? Specifically, what are the effects of proposal power, short-term enforceability of agreements, and uncertainty about future prices?

Our central finding is that conservation agreements will be plagued by a trade-off between the quantity of forest conserved and the distribution of benefits from conservation. This is intuitive. For the forest owner to conserve, it must be adequately compensated for forgoing consumption. Conserving larger stocks of trees requires that the forest owner consume less and that compensation increase commensurately. If the conservationist has decreasing marginal returns to conservation, then they will only be willing to pay for conservation once the forest reaches a sufficiently small size and any attempt to increase the marginal cost of a unit good beyond what its owner would obtain by consuming it raises the marginal costs of conservation and leads to fewer units being conserved. Subsequently, there exists a trade-off between conserving larger forests and providing larger shares of the bargaining surplus from conservation to the developing countries that own them.

[1]This theme is similar to Colgan, Green and Hale (2021) where they discuss how the distributional consequences of asset revaluation affect the politics surrounding climate change cooperation.

Formally, we explore this trade-off by probing how different bargaining arrangements affect the size of the forest preserved and the share of the surplus going to the developing country. Specifically, we vary (i) whether the conservationist or the forest owner has the power to propose agreements and (ii) whether agreements commit the forest owner to conserve or allow them to abscond with the money and consume anyway. This investigation produces three key results. First, the forest owner can never do worse than receive their market value for the good. When the conservationist has the power to propose binding agreements, then they share none of the surplus with the forest owner.

Second, we find that developing countries can ensure that they receive a higher share of the bargaining surplus by allowing for leakage. A common feature in the environmental literature, leakage occurs when conservation efforts only partially restrict consumption thereby allowing actors to circumvent them. In the context of deforestation leakage entails conserving only a subsection of the forest and leaving the remainder unprotected. To study leakage, we consider what happens when the conservationist is only allowed to pay to protect a proper subset of the forest. We show that leakage equates to a strategic environment where the conservationist can only propose nonbinding agreements. To induce the forest owner to conserve under these conditions, the conservationist must increase their payments such that the forest owner is indifferent between defecting from the agreement and not. Consequently, the forest owner benefits from leakage by allowing them to increase their share of the bargaining surplus. Substantively, this result provides a micro-foundation for leakage, suggesting that it serves as a means by which developing countries can extract surplus from wealthy states in conservation agreements. In terms of conservation, this strategic environment produces the worst results of all those we examine.

Third, we show that conservation is maximized when the forest owner has the power to propose agreements regardless of whether agreements are binding or not. Proposal power allows the forest owner to extract a share of the surplus from agreements from the conservationist, incentivizing the forest owner to reach conservation agreements as soon as possible. It also enables the forest owner to coerce the conservationist into accepting agreements early by giving it the ability to threaten the conservationist with worse agreements off the equilibrium path if the conservationist refuses to pay and allows the forest to shrink further. This is possible because the forest owner cannot commit to consuming a tree conditional on rejection of its demands, thereby causing both countries to play mixed strategies - the conservationist mixes as to whether they accept demands and the resource owner mixes in response when deciding whether to consume or not following a rejection. In such an equilibrium the forest owner is indifferent amongst a set of possible demands when accounting for that larger demands are more likely to be rejected, however the conservationist strictly prefers smaller demands allowing the forest owner to condition their future offers on rejection of present ones. Furthermore, the resource owner's share of the surplus from the agreement is the same as that which it extracted under leakage. This is because of the inability of the forest owner to commit to consuming a tree implies that any agreement must leave it indifferent between consuming a tree conditional on rejection of the agreement. Substantively, these results suggest that conservation agreements optimize both the size of the forest and the share of the surplus going to developing countries when they can set the terms of agreements.

Our model builds on the work of Harstad (2016) who introduced the concept of a ”conservation good", whose consumption by its owner imposes an externality on a potential buyer, and then modelled the strategic interactions that occur when the owner tries to either sell or lease the good to the buyer. To motivate his work, Harstad argues that forests are a conservation good and that his model captures the strategic interactions involving payment for ecosystem services. However, forests are large and composed of numerous hectares that cannot all be consumed at once. Therefore, our work extends Harstad (2016) by analyzing the case where the resource owner has numerous units of the conservation good, which they can consume at a limited rate. This allows us to study the trade-off between the number of units conserved, the distribution of benefits from conservation, and how this is impacted by different bargaining protocols, limited commitment, and changing prices. Moreover, certain features of our modelling environment, like leakage, can only be reproduced by studying a market with multiple units of the consumption good.

An additional number of related papers study conservation problems in different contexts where one or more countries want to induce others to conserve a resource.[2] Harstad (2012) studies how countries can combat CO2 emissions when each can only reduce their own consumption of energy, thereby generating leakage - as a country reduces its demand for unclean energy, its price drops and consumption increases elsewhere. He shows that country's can solve the leakage problem by buying coal reserves and then conserving them, thereby reducing the supply of unclean energy inputs. Harstad (2022) studies how to design trade agreements that eliminate a resource owner's incentive to consume a forest in order to transform it into farmland so as to produce more resources to be traded.

[2]A subset of these focuses on issues related to the role of monitoring and illegal deforestation on conservation or the ability to conclude and implement conservation agreements. These are always focused on strategic interactions that occur on the subnational level, either between an NGO and a single community (Gjertsen et al. 2021) or on spillover effects on illegal deforestation between local governments (Burgess et al. 2012; Harstad and Mideksa 2017). However, monitoring does not appear to be an issue for the country-to-country agreements that we study as high-resolution satellite imagery has allowed for an accurate and low-cost solution to the monitoring problem at the national level (Hansen et al. 2013).

He shows that by making tariffs and the division of the gains from trade contingent on the number of trees, a trade partner can eliminate the owner's incentive to conserve. In Section 6 of our paper, we compare this to our qualitatively similar result in which the resource owner induces the interested party to pay to conserve larger quantities of the forest by making the division of the surplus from the agreement contingent on the size of the forest in which an agreement is first reached.

From a technical perspective our model contributes to the theory of bargaining over a good with dynamic value and bargaining with externalities. However, most papers in this vein have the externality arise due to strategic interactions between multiple buyers. For example, Segal (1999) studies a model in which a principal forms contracts with agents that might then impose externalities on agents who do not come to an agreement with the principal. Genicot and Ray (2006) and Iaryczower and Oliveros (2017) consider models in which one or more principals bargain with agents sequentially to contribute to a public good wherein the value to contributing changes with the number of agents who have already joined. Jehiel and Moldovanu (1995) show that delay can arise in the sale of an indivisible good whose purchase imposes negative externalities on other buyers.

We start with a discussion of conservation agreements, and the international framework created to facilitate them. We then define a conservation good, describe the scope of our analysis, and analyze a model of the bargaining between two countries over conservation. Next, we consider a series of extensions of the baseline model where we vary the proposal power, and the short-term enforceability of agreements, uncertainty about future prices. Along the way, we describe how features of international environmental politics motivate our modeling choices and the empirical implications of our theoretical result.

2 Payment for Ecosystem Services Programs

Within the environmental literature and the policy community, agreements wherein one or more countries provide transfers to another to reduce its levels of deforestation are typically categorized as a form of Payment for Ecosystem Services (PES). More broadly, PES programs are defined as "...a transfer of resources between social actors, which aims to create incentives to align individual and/or collective land use decisions with the social interest in the management of natural resources (Muradian et al. 2010, p.1205)." [3] PES programs are attractive in an international context because they provide positive incentives for conservation by allowing those who hold resources that have both economic and

[3] Also see Wunder et al. (2020).

environmental value to be compensated for foregoing the economic benefits. Moreover, their voluntary nature implies no loss of sovereignty or value by resource owners, who are most often developing countries (Angelsen 2017, pp. 248-249).

Recognizing these benefits and the role of deforestation in climate change, the international community sought to promote PES programs targeted at the conservation of forests. At COP 19, held in Warsaw in 2013, the United Nations formally adopted a framework for PES programs called Reducing Emissions from Deforestation and Degradation (REDD+). The Warsaw framework sets forth three important principles for the governance of PES programs.[4] First, countries were to be assessed on their eligibility for payments based on their conservation efforts at the national level. This set the stage for the national government being the principal negotiators of PES programs under REDD+. Second, a reduction of CO2 emissions was adopted as a standardized measure by which program recipients would be assessed. Specifically, a REDD+ program requires an established baseline level of CO2 emissions from forests and then specifies the level of changes from that baseline that will result in payment. Third, it made payment contingent on "measurable, reported, and verifiable results". Though a REDD+ program requires that countries have a national forest monitoring system, satellite imagery allows for quick detection of any large-scale misreporting of tree cover loss (Hansen et al. 2013).

Numerous developing countries have signed or attempted some form of PES program. Norway has led the charge in funding such programs and signed REDD+ agreements with Brazil, Ethiopia, Guyana, Indonesia, Liberia, Myanmar, Mexico, Tanzania, and Vietnam (Angelsen 2017). The success of these REDD+ programs has varied. For example, a bilateral agreement negotiated between Norway and Guyana in 2009 in which Norway promised Guyana 250 million dollars for limiting its deforestation activities is credited with reducing tree cover loss by 34 percent over a five-year period (Roopsind, Sohngen and Brandt 2019). By contrast, a similar 2010 agreement between Norway and Indonesia had limited effects, and very little of the one billion dollars pledged was ever transferred (Williams 2023, Ch. 3).

In other instances, developing countries have put out a call for funds. In May 2022, the Democratic Republic of Congo announced that it would auction rights to oil and gas exploration in large swathes of the rainforest and peatland with the goal of securing resources to further economic development (Maclean and Searcey 2022). Congo expressed a willingness to withdraw the lots from auction if provided with a suitable alternative. However, at the time of this writing, no country or coalition of countries had come forth with a monetary offer to pay the DRC to preserve the lots. Similarly, Ecuador's aforemen-

[4] Voigt and Ferreira (2015) provides an excellent summary of the Warsaw framework.

tioned attempt to receive payments to conserve the Yasuní rainforest ended in failure.

Finally, note that our focus is on country-to-country PES programs. As such, we exclude the many national initiatives that sometimes receive international support. State or NGO led programs, while important, are not covered by the scope of our theory.[5]

3 Baseline Model

Given the typical structure of an international payment for conservation agreement, we start with a baseline model of dynamic two-player bargaining with short-term agreements. Formally, assume there are two players, Home and Foreign, who participate in an infinite-horizon dynamic game that occurs in discrete time $t = 1, 2, 3, \ldots$. At the outset, Home controls N units of a non-renewable resource that they can consume one at a time for as long as there is resource to consume. Foreign has a preference for the resource to be conserved and go unharvested. Examples of such goods are a rainforest or an endangered species. It could also represent a preference not to drill for oil or mine for minerals.

At a time t, there are $N_t \leq N$ parcels of the resource remaining. At the beginning of each period, Foreign proposes a transfer x_t it is willing to make to Home for conservation. Home then can accept or decline the transfer. [6] Initially, we assume that if Home accepts the transfer, then they must conserve that period. Later, we will relax this assumption and allow for agreements that do not commit Home to conserve. For now, define the set of all possible states of the resources as $\mathbb{N}_t = \{0, 1, \ldots, N_t\}$. Then, the action space for Home is

$$A_H = \{(reject, conserve), (reject, consume), (accept)\} \tag{1}$$

and the action space for Foreign is

$$A_F = \mathbb{R}_+.$$

Finally, let $\mathcal{H}$ be the set of histories of the game, including all previous actions and realization of the state, with a specific history at time t being h_t.

A strategy for Home is then a function mapping the state, the offer, and the history into the set of actions. Formally $\sigma_H : \mathbb{N}_t \times \mathbb{R}_+ \times \mathcal{H} \to \Delta A_H$. A strategy for Foreign is a mapping from the state and the history into transfers, $\sigma_F : \mathbb{N}_t \times \mathcal{H} \to \Delta A_F$.

[5] An example of a state-led program would be China's rewarding of farmers for converting farmland to forest along the yellow river to prevent erosion between 1998 and 2005 (Cao, Chen and Yu 2009) (Li 2003). An example of NGO led program would the Carbon Livelihoods Project in Mozambique that offered direct subsidies to farmers who planted trees on their farms (Hegde and Bull 2011).

[6] We model the payments in this way because many REDD+ agreements specify annual payments (conditional on conservation) and, furthermore, sovereign countries are not bound to any long-term agreement they might reach on these issue. See https://www.reddprojectsdatabase.org/view/map.php.

Thus, for every x_t a strategy for Home is a probability for each action.

$$\sigma_H(x_t|N_t,h_t) = \{\sigma_H^a(x_t|N_t,h_t), \sigma_H^{rc}(x_t|N_t,h_t), \sigma_H^{rp}(x_t|N_t,h_t).\} \tag{2}$$

and for each state and history, a strategy for Foreign is an offer (distribution),

$$\sigma_F(N_t|h_t) = F(x_t|N_t,h_t). \tag{3}$$

such that

$$Pr(x_t \leq \hat{x}|N_t,h_t) = F(\hat{x}|N_t,h_t).$$

Home's utility in a period for consuming a unit of resource is π, while not consuming in a period gives a payoff of zero.[7] Foreign has a period utility $u_F(n)$, which we assume to be increasing and concave in the units of the resource remaining at the end of the period, with $u_F(0) = 0$. Foreign also has a disutility of transfers $-x$.

All players discount future payoffs by a common discount factor $\delta \in [0,1]$. Also, let $\psi(N_t, a_H)$ be the transition function for the state and define it such that

$$\psi(N_t,a_H) = \begin{cases} N_t \text{ if } a_H \in \{(accept), (reject, conserve)\} \\ N_t - 1 \text{ otherwise.} \end{cases} \tag{4}$$

Foreign has the following expected utility function

$$\begin{aligned} U_F(x_t|N,h_t) = \sum_{a_H \in A_H} \sigma_H^{a_H}(x_t|N_t,h_t)[u_F(\psi(N_t,a_H)] - \mathbb{1}_{a_H} x_t \\ + \delta V_F(\psi(N_t,a_H)|h_{t+1})] \end{aligned} \tag{5}$$

where $\mathbb{1}_{a_H}$ is an indicator function equaling 1 if Home accepts the transfer. Home's expected utility function for a strategy is given by

$$\begin{aligned} U_H(x_t|N,h_t) = \sigma_H^a(x_t|N_t,h_t)[x_t + \delta V_H(N_t|h_{t+1})] \\ + \sigma_H^{rc}(x_t|N_t,h_t)[\pi + \delta V_H(N_t - 1|h_{t+1})] \\ + \sigma_H^{rp}(x_t|N_t,h_t)\delta V_H(N_t|h_{t+1}). \end{aligned} \tag{6}$$

Our solution concept is subgame perfect equilibrium, meaning that we require that both Foreign's and Home's strategy maximize their expected utility at every subgame.

[7] π can capture the irreversible elements associated with the value of consuming the resource. For example, the market price, but also the value that the conversion of the resource into capital that can have long-run productivity effects.

4 Foreign Makes Offers with Commitment

First we show that any subgame perfect equilibrium where Foreign can make binding offers can only feature agreements as a result of pure stationary Markov strategies. This greatly simplifies the analysis and rules out scenarios where either country punishes or rewards the other for past behavior. For example, Home cannot extort Foreign by rejecting offers and chopping down trees to receive larger transfers from Foreign in future periods. Similarly, Foreign cannot entice Home to delay consuming the forest by promising large future payments.

To begin, note that in any state that the two countries reach an agreement, their utility is fixed and independent of history. All proofs can be found in the appendix.

Proposition 1. *(History-Independent Utilities) In any subgame perfect equilibrium, countries have state-dependent utilities that are independent of history. Homes's is always given by*

$$U_H^*(N) = \sum_{i=1}^{N} \pi\delta^{i-1} \tag{7}$$

and Foreign's utility is given by

$$U_F^*(N) = \frac{u(N)}{1-\delta} - \sum_{i=1}^{N} \pi\delta^{i-1} \tag{8}$$

if the countries are in a state in which Foreign makes an acceptable offer.

The logic underlying this result is straightforward. Though the game has an infinite horizon, the state with zero trees is an absorbing state that provides both countries with a unique payoff of zero. Since this payoff is fixed, Home cannot condition Foreign's future payoffs on their actions when there is one tree. As a consequence, the most Home can expect to receive by cutting down the tree is π. It follows that Home should accept any stream of payments that guarantee it at least π. Since Foreign will seek the minimal possible payment home will accept, Home will receive a stream of payments worth exactly π. But this then implies that both countries will have a fixed utility in the state with just one tree. The proof uses an induction argument to show that this logic generalizes, so that payoffs are fixed in states with any arbitrary number of trees N.

Having demonstrated that the two countries have state utilities that are independent of history in any subgame perfect equilibrium, we can now show that the countries must play pure Markov strategies whenever an agreement is reached. We hereby refer to an equilibrium that features such an agreement as a Pure Strategy Markov Agreement Equilibrium.[8]

Proposition 2. *(Markov strategies) Any subgame-perfect equilibrium must be a pure Strategy Markov Agreement Equilibrium.*

To see why this is true, first observe that in states where an offer is accepted with positive probability a pure Markov strategies require that Foreign make a state-dependent offer $x^*(N)$ that guarantees Home a utility of $U_H^*(N)$, which is history-independent, to sustain conservation over time. Any deviation from this payment that also makes Home indifferent between accepting and rejecting and consuming, but depends on the history, must be balanced out with either higher or lower payments in future periods. The problem is that one of the two players would always have an incentive to deviate. If payments are ever larger than $x^*(N)$, then Foreign can always reduce the size of their payment and have Home accept in that period. Conversely, if Foreign ever makes an offer smaller than $x^*(N)$, then Home will strictly prefer to chop down the tree rather than accept the offer. With utilities fixed for future states, the two countries are unable to adjust their future strategies to prevent these deviations; Foreign is therefore unable to commit to making higher offers in the future and Home is unable to commit to accepting low offers. We can also rule out equilibria where Foreign plays mixed strategies when making offers with a mean of $x^*(N)$ because at any history where the realization of the offer is less than $x^*(N)$ Home prefers to consume. This eliminates the possibility of the countries making history-dependent offers, 1) when there is a probability of accepting and 2) that put positive probability on smaller offers, but ensure Home an expected utility of $U_H^*(N)$.

Substantively, this means typical mechanisms associated with repeated games common in the environmental politics literature have limited application for payment for ecosystem service problems (Barrett 2005). Punishment and reward strategies that depend on historical reputations for conservation or consumption are not particularly relevant for successful cooperation in this instance.

4.1 Optimal Offers

The previous section demonstrated that both countries play pure Markov strategies when agreements are reached. This implies that we can focus on strategies that depend solely

[8]Note that strategies in this case are not fully Markov strategies because, when no agreement can be reached, Foreign can play any strategy conditional on its offer being rejected. If it doesn't matter what low offer, Foreign makes, it could, in principle, make lowball offers that are history-dependent.

on the current size of the forest N_t. The following Proposition characterizes the payments required to conserve the forest.

Proposition 3. *If the two countries reach an agreement in the state with N trees, then Foreign will make the following offer*

$$x^*(N_t) = (1-\delta)\sum_{i=1}^{N} \pi\delta^{i-1} = \pi(1-\delta^N) \tag{9}$$

Interpreting these payments is straightforward. Though Home would only obtain π from chopping down a section of the forest in this period, Home can anticipate the returns of future consumption. Therefore, Foreign must compensate Home for the discounted stream of π values it would receive if it consumed the entire forest.

Note that δ exerts two competing effects on the size of payments. First, when δ is small, such that Home discounts the future, Home's value for the forest $\sum_{i=1}^{N} \pi\delta^{i-1}$ decreases. Because Home can only consume one tree at a time, Home will have a lower value for trees that she will only be able to consume in future periods. Conversely, as δ increases Home's value for the forest increases, and she will demand to be paid not only for the tree she will conserve today, but also for her future consumption options.

Second, when δ is small, Home does not value the promise of future payments and requires a larger portion of her value of the forest to paid in the current period. By contrast, when δ is large, Home values future payments more, thereby allowing Foreign to make the payments in smaller installments. This second effect is captured in the $(1-\delta)$ term.

The following Corollary demonstrates that the second effect on δ dominates the first. Specifically, it shows that $x^*(N)$ is strictly decreasing in δ and provides upper and lower bounds on the size of payments as a function of both δ and N.

Corollary 1. $x^*(N)$ *satisfies the following*

(i) $\lim_{N\to\infty} x^*(N) = \pi$

(ii) $\lim_{\delta\to 0} x^*(N) = \pi$

(iii) $\lim_{\delta\to 1} x^*(N) = 0$

Here we see two surprising results. First, Foreign's per period payments are capped at π, even as the forest grows to an arbitrarily large size. When Home can expect to chop down trees forever, they must be given the full value of a tree every period so as not to consume in every period. This result can help explain why we observe that many real-life payments for ecological services seem so paltry. Second, this corollary demonstrates that Home's utility doesn't always correlate with the size of payments. Though both payments and Home's utility are weakly increasing in the number of trees, Home's utility strictly increases with the discount factor even as payments decrease. As the limit of δ goes to 1, payments shrink to 0, but Home's utility increases to $N\pi$. By contrast, when the limit of δ goes to 0, payments increase to π, but Home's utility decreases to π.

Regardless of the value of δ or N, Home does not benefit when bargaining with Foreign making offers. Though conservation generates a surplus, this remains entirely with Foreign as Foreign makes the minimal required payments to guarantee Home their utility from the consumption of trees. As a result, Home's expected utility from conservation is always equal to their market value for the forest.

4.2 Equilibrium Size of the Forest

Because the states play Markov strategies whenever an agreement is reached, once the forest reaches a size in which Foreign chooses to pay for conservation, then the game enters a steady state. There exists a unique cutoff number of trees N^* such that Foreign will always make an offer whenever the size of the forest is at or below that size. If at time $t = 0$, $N < N^*$, then Foreign will immediately begin by making an acceptable offer and no trees will ever be chopped down.

On the other hand, if at time $t = 0$, $N > N^*$, then the game will feature $N - N^*$ periods during which Foreign will not make acceptable offers and Home will choose to chop down a tree. However, once Home has chopped enough trees so that the game reaches N^*, Home will always choose to make acceptable offers. At this point, the game will enter a steady state, and the Forest will remain at size N^* in perpetuity. The following Proposition formalizes this argument.

Proposition 4. *(Forest Size I) There is a unique N^* such that Foreign will never make acceptable offers in a period with $N_t > N^*$ and will always make an offer in states with $N_t \leq N^*$. N^* is given by*

$$\max_n \{ \frac{u_F(n) - u_F(n-1)}{1-\delta} \geq \pi \} \tag{10}$$

If no such n exists, then $N^* = 0$ and Foreign never issues an acceptable offer $x^*(N)$.

The following is the interpretation of this result. When the forest grows large, Foreign's marginal benefit from conserving an additional tree, represented by $\frac{u(N)-u(N-1)}{1-\delta}$, decreases as the forest increases in size while the marginal cost of choosing to preserve

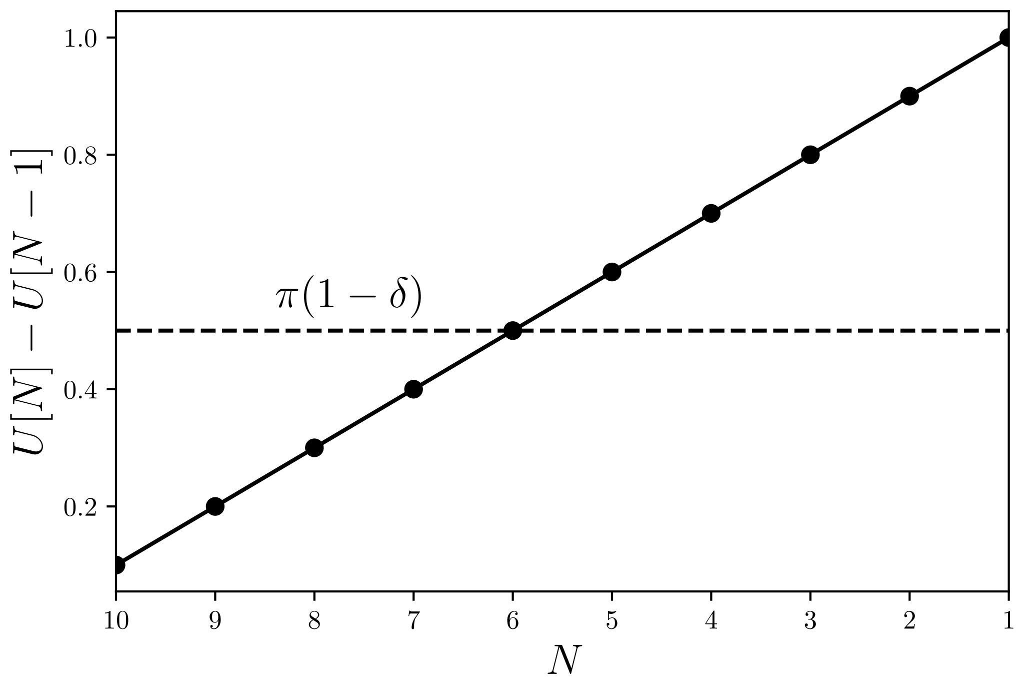

Figure 1. Decreasing Marginal Benefits to Conservation: This figure depicts Foreign's increasing marginal utility to conservation as the forest shrinks in size. The dashed line represents the constant marginal cost of choosing to preserve an additional tree in perpetuity. Foreign will act to conserve all trees at or above the line, and will Home to consume the Forest otherwise. Note that the solid and dashed lines need not intersect at an integer.

the forest in the current period instead of the following period is constant and is represented by π, the amount that Home must be paid to preserve a tree. It follows that if the marginal benefits to conserving an additional tree exceed the benefits of the current tree count, then it must be justifiable to conserve the forest for any smaller tree count. Figures 1 and 2 provide an illustration of the intuition underlying these results.[9]

5 Relaxing Commitment

One might worry that we have assumed too much commitment on Home's side. Even though we only allow for one-period agreements, it is interesting to know what happens when even these short-term agreements must be self-enforcing. Therefore, we now relax the assumption that Foreign can make binding offers and allow Home to collect Foreign's

[9]In both figures, $\pi = 10$, $\delta = 0.95$, and $u(N) = u(N-1) + (1 - 0.1N)$ with $u(0) = 0$.

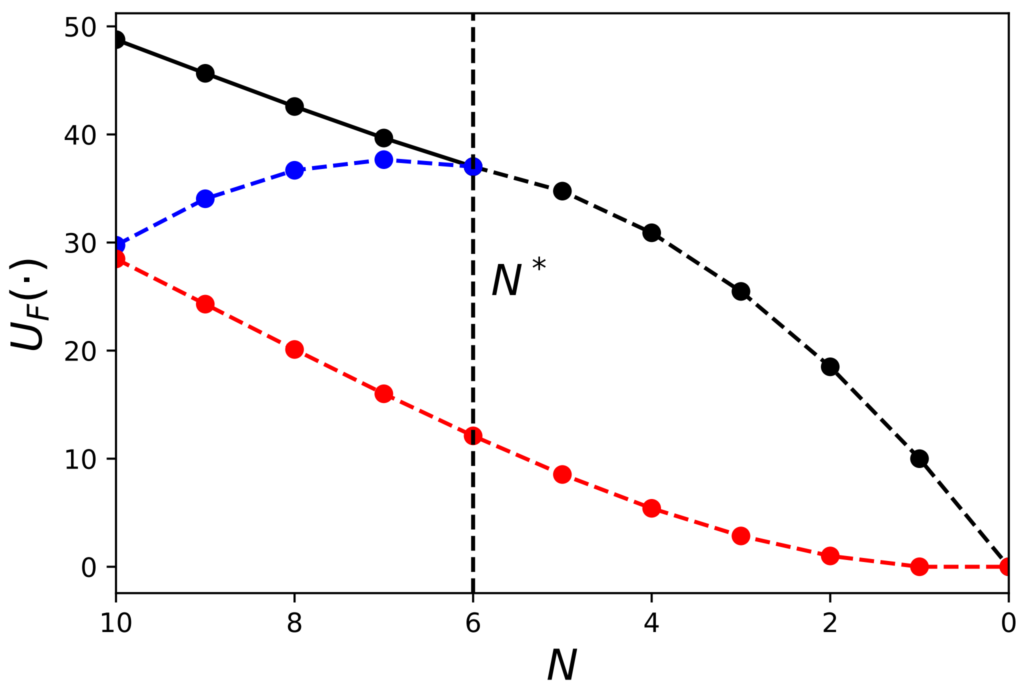

Figure 2. Equilibrium Utility: This figure depicts the utility to Foreign from taking various actions. The red line represents Foreign's utility from allowing Home to consume the forest as the forest shrinks in size. The blue line represents Foreign's utility from conserving the forest at different at each of the different possible sizes. The black line represents the equilibrium path. Foreign does nothing, allowing Home to consume at all tree counts to the left of the vertical dashed line and pays Home to conserve at all tree counts at or to the right of the dashed line. The black line is dashed to the right of the dashed line to emphasize that this off-the-equilibrium path because conservation of the story at N^* is a steady state.

payment before deciding whether to chop down the tree. We show that countries can continue to reach agreements to conserve the forest. Moreover, equilibrium behavior remains qualitatively similar, with both countries playing Markov strategies whenever an agreement is reached so that conservation is a steady state. However, Home can leverage its ability to defect to extract a greater share of the surplus from conservation from Foreign. As a result, Foreign will conserve smaller forests.

Proposition 5. *(No Commitment) There exists a unique subgame perfect equilibrium that is a pure strategy Markov agreement equilibrium. If the two countries come to an agreement, then Foreign will offer Home*

$$x^{**}(N) = (1-\delta)\frac{N\pi}{\delta} \tag{11}$$

and Home will accept and conserve the forest. Foreign's utility from and agreement will be given by

$$U_F^{**} = \frac{u(n)}{1-\delta} - \frac{N\pi}{\delta} \tag{12}$$

and Home's utility will be given by

$$U_H^{**} = \frac{N\pi}{\delta} \tag{13}$$

The intuition that explains this result is similar to that underlying the commitment case. The state with zero trees continues to be an absorbing state that provides both states with a unique payoff of zero. When Home threatens to defect on agreements in the state with one tree, Home can expect to receive π plus whatever payment $x(1)$ made in that period. It follows that Home should conserve when it expects a stream of payments that is large enough to exceed the one-time payoff from defecting. Since Foreign will seek to make the minimum payment required to prevent Home from chopping, Home will receive a stream of payments that leaves it indifferent between chopping down the tree and absconding with Foreign's payment. This implies that countries have a fixed utility in the case with one tree. As before, the proof uses an induction argument to generalize the argument to an arbitrary number of trees N. Moreover, Foreign's inability to commit to making large offers in future periods rules out history-dependent strategies and requires that the countries play pure Markov strategies whenever an agreement is reached.

Foreign's payment to Home is then constructed by choosing a payment $x(N)$ such that the stream of payments it produces if accepted is at least as good as that from accepting the payment, cutting down the tree, and moving to a state with $N-1$ trees. In the state where $N = 1$, this implies $x(1) = (1-\delta)\frac{\pi}{\delta}$. In turn, in the state with two trees, Home must be indifferent between taking the payment, consuming a tree π, and moving to the state with one tree for a utility of $\delta\frac{\pi}{\delta}$ or conserving and receiving $\delta\frac{x(2)}{1-\delta}$. This implies that $x(2) = (1-\delta)\frac{2\pi}{\delta}$ is the smallest offer Foreign can make to achieve conservation. Equation 11 is a generalization of this argument to the case with N trees.

5.1 Implications for Payments

The presence of commitment problems presents two important substantive implications. First, conservation becomes much more difficult. The following corollary presents how comparative statics related to payments change as a result

Corollary 2. *$x^{**}(N)$ satisfies the following*

(i) $\lim\limits_{N\to\infty} x^{**}(N) = \infty$

(ii) $\lim\limits_{\delta\to 0} x^{**}(N) = \infty$

(iii) $\lim\limits_{\delta\to 1} x^{**}(N) = 0$

Whereas Corollary 1 showed that payments for conservation under binding agreements were capped at π, the result above demonstrates there is no upper bound for the payments Home requires to conserve with nonbinding agreements. The costs of preserving large forests scales linearly with the cost of the forest and can grow infinitely large. Moreover, payments to highly impatient states can require arbitrarily large sums to achieve conservation, even when the number of trees is small. However, as Home becomes more patient the per period payment required to induce it to conserve the forest remains small.

Second, as Foreign's payments increase, so does Home's utility from agreements. The following corollary provides a formal statement of this result.

Corollary 3. *When agreements do not commit Home to conserve, Home's utility from conservation increases by*

$$\Delta(N,\delta) \equiv \frac{\pi N}{\delta} - \sum_{i=1}^{N} \pi\delta^{i-1} \tag{14}$$

which is strictly positive for all N and $\delta < 1$ and equals zero for $\delta = 1$.

The intuition for this result is simply that higher per period payments imply a higher utility for Home, except for the case where $\delta = 1$ since in both cases payments converge towards zero as $\delta \to 1$. Because binding agreements only ever gave Home their market valuation for the forest, it must be the case that any increase in utility that occurs with nonbinding agreements takes place because Foreign is splitting the surplus from conservation with Home.

5.2 Equilibrium Size of the Forest

Increased payments for Home raise the cost of conservation and lead to smaller forests being preserved. As in the commitment case, the fact that countries play Markov strategies whenever an agreement is reached implies that conservation is a steady state - if Foreign is willing to make an offer that Home will accept and abide, then they will always do so. Once again, this implies that there exists a unique N, denoted N^{**}, such that Foreign will make acceptable offers if and only if $N_t \leq N^{**}$. The following Proposition formalizes this argument and provides a solution for N^{**} that is directly comparable to N^*.

Proposition 6. *(Forest Size II) There is a unique N^{**} such that Foreign will never make acceptable offers in a period with $N_t > N^{**}$ and will always make an offer in states with $N_t \leq N^{**}$. N^* is given by*

$$\max_n \{ \frac{u_F(n) - u_F(n-1)}{1-\delta} \geq \frac{N\pi}{\delta} - (N-1)\pi) \} \tag{15}$$

If no such n exists, then $N^{**} = 0$ and Foreign never issues an acceptable offer $x^{**}(N)$.

The interpretation of this condition is similar to that of equation 10. The sole difference is that the marginal cost of conservation has increased from π to $\frac{N\pi}{\delta} - (N-1)\pi$. It is easy to check that this value is greater than π for all values of $\delta < 1$. As demonstrated by Corollary 3, this is because Foreign's payments to Home increase for all $\delta < 1$. This leads to the following result.

Corollary 4. $N^* \geq N^{**}$

Intuitively, as the marginal cost of a tree increases, the number of trees Foreign will be willing to pay to conserve must (weakly) decrease.

5.3 Leakage as Partial Commitment and Enforcement

Payment for ecosystem services is often predicated on public land holdings. However, the amount of public and private forests and other resources varies from country to country; in the DRC nearly 100% of forests are public (*Forest Resources and Context of Democratic Republic of the Congo* 2020), while in the United States, 60% of forests are privately owned and manged (*Forest Ownership Statistics* 2018). The transfer of public land to private management can limit a government's ability to commit to international conservation agreements.

In this section, we show that a model of conservation with the potential for leakage - where Home can only commit to conserving a percentage of the forest, and the remainder is available for Home to consume - is equivalent to a model of bargaining without commitment. We refer to units that Foreign has contracted to conserve as protected and the remainder as unprotected. Our results show that (i) conservation is possible even when some trees remain unprotected, (ii) that Proposition 5 characterizes agreements when trees remain unprotected, (iii) and that Proposition 6 again describes the number of trees protected. By contrast, when Foreign is able to protect the entire forest, then the agreements reached are the same as those in Section 4 where Foreign made offers with commitment. Consequently, Home has strong incentives to ensure that any conservation agreement is plagued by the potential for leakage.

Formally, we now assume that an offer from Foreign consists of two dimensions: a transfer $x_t \in \mathbb{R}_+$ and the number of trees to protect $Q \in \{0, 1, ..., Q(\bar{N}_t)\}$ where $\bar{Q}(N_t) \leq N_t$. Furthermore, we assume for now that $\bar{Q}(N_t)$ is exogenous and that the potential for leakage either exists $\bar{Q}(N_t) < N_t$ for all t or it does not $\bar{Q}(N_t) = N_t$ for all t. Home's action space is contingent on $Q(N_t)$

$$A_H = \begin{cases} \{\text{accept, reject}\} \times \{\text{consume, conserve}\} & \text{if } Q_t < N_t \\ \{\text{accept, conserve}\}, \{\text{reject, consume}\}, \{\text{reject, conserve}\} & \text{if } Q_t = N_t \end{cases} \tag{16}$$

The following Proposition provides a formal statement of the result

Proposition 7. *(Leakage) There exists a unique subgame perfect equilibrium that is a pure strategy Markov agreement equilibrium:*

(i) If $\bar{Q}(N_t) = N_t$ and $N_t \leq N^*$ as defined in equation 10, then Foreign sets $Q(N_t) = N_t$ and offers Home a transfer of x^* as defined in equation 9.

(ii) If $\bar{Q}(N_t) < N_t$ and $N_t \leq N^{**}$ as defined in equation 15, then for any $Q(N_t)$ Home offers Foreign a transfer of x^* as defined in equation 11.

(iii) Otherwise, no agreement can be reached in which Foreign sets $x(N) > 0$ and Home consumes.

The following is the intuition underlying the result. The potential for leakage $\bar{Q}(N_t) < N_t$ for all t is equivalent to the game with no commitment. However, when $\bar{Q}(N_t) = N_t$, it could theoretically be possible for Home to reject demands and consume when Foreign sets $\bar{Q}(N_t) = N_t$ in an attempt to induce Foreign to only make the nonbinding offers that would increase Home's utility. The crux of the proof is to show that Home stands to gain nothing by rejecting such an offer because nonbinding agreements were already designed to keep Home indifferent between consuming a tree in the current period and waiting for an offer in the next period. For example, consider the case with one tree. Recall from Proposition 5 that with nonbinding agreements, Home can extract a stream of payments that sum to a value of $\frac{\pi}{\delta}$ from Foreign. Clearly, Home is indifferent between consuming the tree and waiting a period to receive that stream of payments. However, that must mean that Home would also be willing to accept a stream of payments starting in the current period that guarantee it a utility of π. The proof extends this logic to N trees using an inductive argument.

As the above discussion makes clear, Home's utility is strictly lower when Foreign is allowed to offer conservation agreements for the entire forest. However, this does not necessarily mean that Home has had no recourse other than to accept an offer from Foreign that binds it to conservation. As a sovereign state, Home may be able to exclude certain sections of a forest from inclusion in agreements by handing over their ownership to a cooperative private owner or other means. Therefore, it is possible that Home can set $\bar{Q}(N_t)$. This leads to the following Corollary.

Corollary 5. *If Home can set $\bar{Q}(N_t)$, then they will always select $\bar{Q}(N_t) < N_t$*

This result suggests that leakage may be an endogenous institutional feature of conservation, designed by the owner to increase its share of the surplus from cooperation.

6 Home Setting Payment

Leakage is not the only means by which developing countries can seek to increase their share of the bargaining surplus. Recently, Brazil, Indonesia, who together own more than 50 percent of the world's tropical rainforests, have launched talks to form a coalition of rainforest owning countries (Greenfield 2022). Dubbed the "OPEC of rainforests", this coalition is aimed at increasing their bargaining power in the international negotiations on climate change and conservation and attaining more financial support from the rich nations for preserving their forests. What happens when the resource rich countries take the initiative in bargaining for conservation?

To answer this question, now consider how the game changes when Home is allowed to make demands instead of Foreign. The structure of the stage game is now as follows: Home makes a demand x of Foreign that Foreign must pay for Home to conserve the Forest. If Foreign accepts the demand, then it pays a transfer x to Home. As in the previous section, we assume that Foreign cannot ensure that Home will be committed to the agreement. If Foreign rejects Home's offer, then Home can then choose whether to chop down a section of the Forest. Under this protocol, Home's utility is equal to that which it achieved under nonbinding agreements, though Foreign's is always weakly smaller.

We begin by restricting our attention to Markov Perfect Equilibria in mixed strategies.[10] Such equilibria feature Home making a demand, Foreign accepting the demand with positive probability, and then Home chopping down the forest with positive probability if its demand is rejected. Let $p(x,n)$ and $q(x,n)$ respectively denote the probability that Foreign accepts an offer and the probability that Home chops conditional on a rejection. The following Lemma characterizes the latter two strategies and both countries' expected utilities as a function of the number of trees and Home's demand x.

Lemma 1. *(Mixture) In any mixed strategy Markov equilibrium:*

(i) $p(x,N) = \frac{N\pi(1-\delta)}{x\delta}$

(ii) $q(x,N) = \frac{x(N)(1-\delta)}{u(N)-\delta x-(1-\delta)[u(N-1)+\delta V(N-1)]}$

(iii) $U_H(N)$ *is given by equation 13:* $U_H^{**} = \frac{N\pi}{\delta}$

(iv) $U_F(x,N)$ *is given by* $\frac{u(n)-x}{1-\delta}$

(v) $x \in \left[\frac{N\pi(1-\delta)}{\delta}, u(n) - (1-\delta)\sum_{i=2}^{N} u(i-1)\delta^{N-i}\right]$

The intuition underlying the result is straightforward. Conditional on a demand being rejected, Home mixes over whether to consume a tree in the present period or wait to reissue the demand in the next period. The larger the demand, the more tempting it is to wait to reissue it. To ensure that Home is indifferent between chopping and not, Foreign accepts higher demands with lower probabilities. Regardless of the size of the demand, at the start of every period, before Home learns whether or not Foreign will accept its

[10]The online appendix discusses the robustness of this assumption. Following Harstad (2016), we apply a renegotiation proofness refinement and show that the only equilibria that survive retain the key qualitative features of Markov Perfect Equilibria we characterize. Namely, we show that (i) agreements can only be reached when the countries play mixed strategies, (ii) Home's utility from agreements remains the same, and (iii) the set of states (i.e. the number of trees) for which an agreement is reached with positive probability is identical regardless of whether an equilibrium is in Markov strategies or not.

demand, it has a utility of $\frac{\pi}{\delta}$. This is the same amount that Home received when Foreign had proposal power but could not propose binding agreements.

As before, this larger surplus comes at Foreign's expense. How much worse off Foreign is depends on the demands that Home issues in each state - since Foreign must be indifferent as to whether or not it accepts a demand, its utility can be expressed as

$$U_F^*(x|N) = \frac{u(n) - x}{1 - \delta} \tag{17}$$

Clearly, the higher the demand, the lower Foreign's utility. The highest possible demand Home can make leaves Foreign with none of the bargaining surplus. Foreign accepts the offer with a low probability, and Home chops with probability 1 if its offer is refused. The lowest possible demand that Home can make is the minimal possible demand is that which allows its utility to sum up to $U_H^{**}(N)$ if Foreign were to accept that offer every period with probability 1. This is the same demand Foreign makes when it has proposal power and can issue non-binding agreements. It follows that Foreign's utility when Home has proposal power must be weakly smaller than its utility in equation 12 when Home had proposal power and could issue nonbinding agreements.

There are strong normative reasons in favor of Home making smaller demands. First, Foreign's utility is strictly decreasing in the size of Home's demand. Second, a lower utility for Foreign at any state is more likely to lead to a smaller forest. This is because Home is more likely to consume a tree when it makes large demands to balance against Foreign's incentive to free-ride and avoid making payments. Third, Home's utility U_H^{**} is independent of the size of its demands, implying that the downsides of larger demands are not offset by Home receiving a larger share of the bargaining surplus. However, there is no guarantee that the countries will play an equilibrium that maximizes Foreign's utility and leads to steady-state conservation.

6.1 Optimal X and the Size of the Forest

Similarly to when Foreign was making offers, the equilibrium in Markov strategies generates a unique cutoff N_H^* such that conservation is only achieved with positive probability at several trees $N \leq N_H^*$. If the game begins at any $N > N_H^*$ no agreement is reached and Home consumes a tree each period. Once a sufficient number of trees is consumed and the state becomes N_H^*, or alternatively, if the game begins at an $N \leq N_*^H$, then Foreign can make demands that Foreign will accept with positive probability. However, unlike the case when Foreign was making offers, there is no guarantee that the game will enter a steady state. This is because for virtually any set of parameters, there is a range of demands Home can make and for which all but the lower bound can be rejected and lead to Home consuming with positive probability. The following Proposition provides a formal characterization of this result.

Proposition 8. *(Forest Size III) There exists a unique N_H^* such that Home will never make an acceptable offer in a period in which $N_t > N_H^*$. At any $N \leq N_H^*$, Home will make a demand from the range $[\frac{N\pi(1-\delta)}{\delta}, u_F(n) - (1-\delta)\sum_{i=2}^{N} u_F(i-1)\delta^{N-i}]$ and will have Foreign accept the demand with positive probability. N_H^* is given by*

$$\max_n \{\frac{u_F(n)}{1-\delta} - \sum_{i=2}^{N} u_F(i-1)\delta^{N-i} \geq \frac{n\pi}{\delta}\} \tag{18}$$

This condition is the first time that accepting and locking in payments and forest size is better for Foreign than just letting Home consumer the whole forest. The following is the intuition underlying the result. Absent any agreements, Home's utility for consuming the forest is given by $\sum_{i=1}^{N} \pi\delta^{i-1}$. However, if an agreement is reached in the state with N trees, then Home's utility increase to $U_H^{**}(N)$. Therefore, Home benefits from reaching an agreement with as large a number of trees as possible. Since Home's utility is independent of x, Home makes small demands that leave Foreign with a surplus of utility from conservation. N_H^* is the largest number of trees at which Foreign's utility from an agreement is larger than their utility stream from allowing Home to consume the entire forest. This implies that conservation is achieved and a deal reached as early in the game as possible.

This is not Foreign's preferred outcome. Without loss of generality, consider Foreign's equilibrium utility as defined in equation 12 at the lower bound of $x(N)$ (as given by 12). Foreign's utility is concave in N, the marginal benefits of conservation $\frac{u(N)}{1-\delta}$ are decreasing in N, while the costs of conservation $\frac{N\pi}{\delta}$ increase at a constant rate. It follows that Foreign's utility could be increased by delaying agreements and accepting the minimal demand at state $N^{**} \leq N_H^*$ after allowing Home to consume additional trees. Figure 3 illustrates this point.

However, Home can induce Foreign to come to an agreement at an earlier date. Though Foreign would like to receive the same low demand $x(N) = \frac{N\pi(1-\delta)}{\delta}$ at $\hat{N}^{**}$, Home is free to make larger demands at lower levels of trees. Specifically, Home can always select a demand that ensures that Foreign does no better from rejecting a demand at N_H^*. Home can credibly commit to this strategy since its utility is independent of x. As a result, Foreign cannot improve their utility by rejecting demands at N_H^* and this off-path "punishment strategy" is never achieved in arrived at. Figure 4 depicts the equilibrium path utilities with the off-path utilities if Foreign rejects Home's demand at N_H^*.

This result has a similar intuition to that in Harstad (2022). In that paper, Harstad shows that if the division of the surplus from trade is conditioned on the stock of trees remaining, then it is possible for free trade agreements to disincentivize the transformation of forests into arable land necessary for the production of a trade good. The recurrence of this logic is somewhat surprising given the different setups of the models - in Harstad (2022) it is the conservationist who conditions payments on the stock, whereas in our result it is the resource owner conditioning on the stock to incentivize the conservationist to reach agreements earlier. The fact that the strategic logic in both models operates similarly despite different structures, likely implies that this is a robust feature of conservation agreements.

7 Markov Equilibrium with Price Shocks

In May 2022, the DRC announced that it would begin to auction large swathes of rainforest and peatland for gas and oil exploration. This represented a departure from the DRC's previous policy, which had promoted conservation and attracted large financial commitments from international actors. However, the Russo-Ukrainan war, launched two months earlier, led to sharp increases in energy prices and prompted the DRC to reconsider its pro-environmental policies (Maclean and Searcey 2022; Wong 2022).

In this section, we consider how the equilibrium of the PES game changes in response to price shocks. To do so, we return to the model in section 4 where Foreign makes offers that commit Home and introduce a price shock at the start of every period. We assume that the price shock takes the form of a publicly observable value for π_t that is independently drawn from the set $\{\bar{\pi}, \underline{\pi}\}$ where $\bar{\pi} > \underline{\pi}$. Let p and $1-p$ respectively denote the probability that $\underline{\pi}(N)$ or $\bar{\pi}(N)$ is drawn. We show that the equilibrium remains qualitatively similar to the case with constant prices. The only exception is that there can now be multiple thresholds N^* for different price points, a $N^*(\underline{\pi})$ for when the price is low, and smaller $N^*(\bar{\pi})$ for when the price is high. Though consumption will slow once the forest reaches $N^*(\underline{\pi})$, it will not stop shrinking until the Forest reaches size $N^*(\bar{\pi})$. As a result, shifting prices lead to smaller forests than does a constant price with average $\pi = p\underline{\pi} + (1-p)\bar{\pi}$.

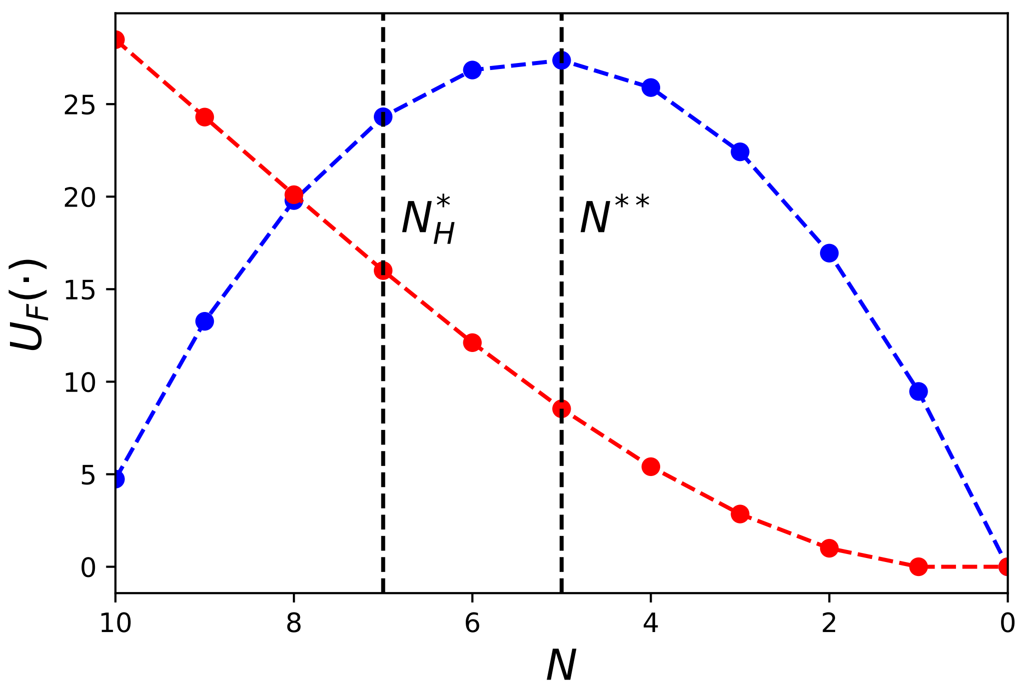

Figure 3. Foreign's Preferences over Agreements: The red depicts Foreign's utility for allowing Home to consume the entire forest. It also represents Foreign's utility for accepting the maximal demand made by Home. The blue line represents Home's utility when Home makes the minimal offer and the entire forest is preserved. Foreign's utility from conservation is concave in the number of trees, implying that Foreign will prefer to conserve at lower forest counts. This is represented by N^{**}. However, Home prefers to reach an agreement as soon as possible and can credibly commit to demand the entire surplus in the future. Therefore, an agreement is reached as soon as the blue line goes above the red one at N_H^*.

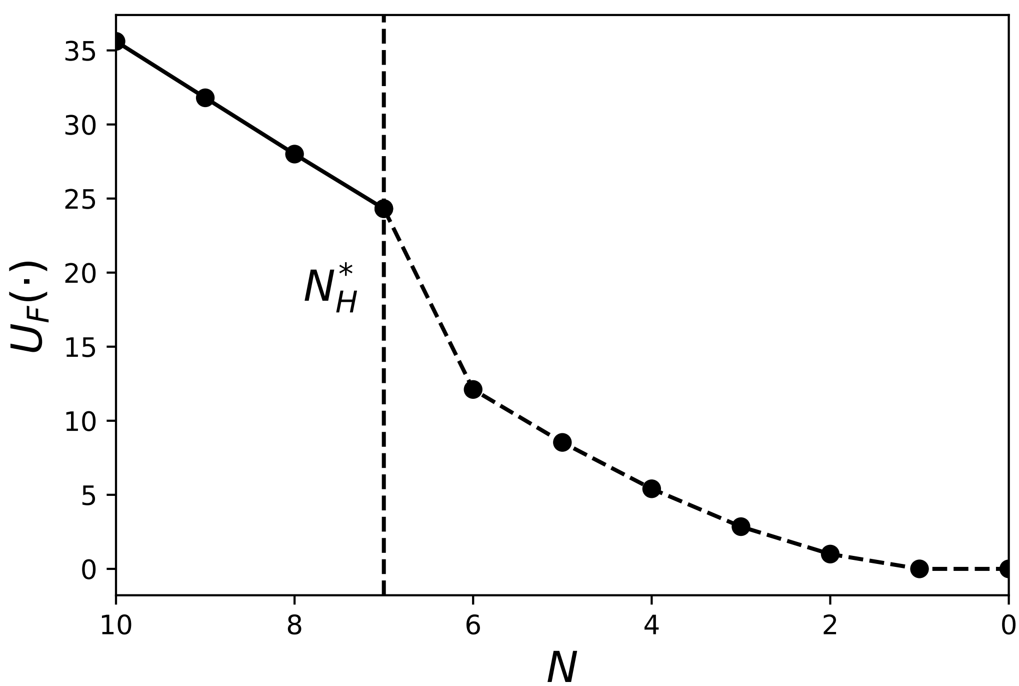

Figure 4. The Equilibrium Path: This figure depicts Foreign's utility at each stage of the equilibrium path. Initially, Home cannot make a demand that Foreign would accept and consumes each period. With 7 trees, Home makes the lowest demand it can and Foreign accepts and the game enters a steady state. This is supported by the off-path behavior to the right of the dashed line, in which Home makes the maximal demand it can of Foreign.

7.1 Equilibrium Strategies

Equilibrium strategies remain similar to the model with constant prices. Any subgame-perfect equilibrium must be a pure startegy markov agreement equilibrium where agreements feature Foreign making the minimum offer each period that keeps Home just indifferent between chopping and not, and Home receives none of the surplus from conservation. There are, however, a few minor adjustments that need to be made to account for the shifting prices.

First, in the model with constant prices, Home always chose to consume absent a payment since there was never any benefit to delay. With price shocks, Home may choose to delay consumption in periods in which $\pi_t = \underline{\pi}$ to await periods with higher prices for trees. The following Lemma defines the conditions under which Home will choose to consume a tree absent any positive offer from Foreign

Lemma 2. *If Foreign does not make positive offers in a given state with N trees, then Home will consume the forest when $\pi_t = \underline{\pi}$ if and only if*

$$\underline{\pi} \geq \frac{\delta(1-p)\bar{\pi}}{1-\delta p} \tag{19}$$

The lemma shows that Home will delay consumption in periods in which $\pi_t = \underline{\pi}$ whenever doing so will be beneficial. The left-hand side represents the benefits of consumption of the tree in the current period. The right-hand side of equation 19 represents the benefit of delaying consumption until the $\pi_t = \bar{\pi}$.

Second, strategies are made conditional on the current value of π. Let $x(\pi_t, N)$ denote the offer that Foreign makes in a period with price π_t and N trees. The following lemma states that Foreign will never provide Home with more than is necessary to consume in the current period. Since Foreign does not consume in periods in which condition 19 does not hold, Foreign will make no payment in such periods. As a result, Home's utilities remain unaffected by whether the parties reach an agreement.

Lemma 3. *If the condition in equation 19 is satisfied, then*

(i) Foreign will either set $x(\pi_t, N) = 0$ and Home will consume the forest or Foreign will set

$$x(\pi_t, N) = \pi_t - \delta^N[p\underline{\pi} + (1-p)\bar{\pi}] \tag{20}$$

and Home will accept the offer.

(ii) Home's utility for being in the state with N trees is given by

$$V_H(N) = \sum_{i=1}^{N} \delta^{i-1} [p\underline{\pi} + (1-p)\bar{\pi}] \tag{21}$$

prior to the realization of π_t

If the condition in equation 19 does not hold, then conservation requires

(iii) Foreign will always set $x(\underline{\pi}, N) = 0$ and Home will not consume.

(iv) If $\pi_t = \bar{\pi}$, then Foreign will either set $x(\bar{\pi}, N) = 0$ and allow Home to consume or Foreign will set

$$x(\bar{\pi}, N) = (1-\delta) \sum_{i=1}^{N} \frac{\delta^{i-1}(1-p)^{i-1}\bar{\pi}}{[1-\delta p]^i} \tag{22}$$

and Home will accept the offer.

(v) Home's utility for being in the state with N trees is given by

$$V(N) = \sum_{i=1}^{N} \delta^{i-1} \left[\frac{1-p}{1-\delta p}\right]^i \bar{\pi} \tag{23}$$

prior to the realization of π_t.

The following is the intuition underlying the result. As in the case with constant prices, Foreign can never commit to paying any more than the minimum required for Home not to consume the forest. The proof demonstrates that any strategy that relies on Foreign attempting to delay payments in periods in which $\pi_t = \underline{\pi}$ (or $\pi_t = \bar{\pi}$) by making smaller offers than those stipulated in equations 20 or 22 and then increasing payments in periods in which $\pi_t = \bar{\pi}$ (or $\pi_t = \underline{\pi}$) do not survive the one-shot deviation principle. Simply put, Foreign cannot commit to paying higher prices than those listed in equations 20 or 22 and would simply deviate to those prices at the earliest possible opportunity. Home will always accept these lower offers, thereby leaving Foreign strictly better off. This implies that the only subgame-perfect equilibrium strategy for periods in which an agreement is reached is a Markov strategy in which Foreign either makes the minimum payment in that period to prevent Home from chopping or does not pay at all.

7.2 Size of the Forest

As in the case with constant prices because the two countries play Markov strategies whenever an agreement is reached, once Foreign is willing to pay to conserve the forest in a certain state, she will always be willing to do so. However, because the state now depends on both the number of trees and the value of π_t, conservation is no longer necessarily a steady state: Foreign may be willing to preserve in a state with N trees if $\pi_t = \underline{\pi}$ but not if $\pi_t = \bar{\pi}$. As a result, there exist two thresholds $N^*(\bar{\pi}$ and the weakly larger $\underline{\pi})$ that give the game a clear equilibrium trajectory. Whenever $N > N^*(\underline{\pi})$, Foreign will be unwilling to make offers capable of conserving the forest regardless of the value of π_t. However, once N enters the range of $(N^*(\bar{\pi}, \underline{\pi})]$, then Foreign will be willing to conserve the forest only in states in which $\pi_t = \underline{\pi}$ and allow Home to consume otherwise.[11] Finally, in any state in which $N \leq N^*(\bar{\pi})$, Foreign will always make an offer that Home will accept for any value of π_t and the forest will enter a steady state.

The following Proposition provides a formal characterization of this result.

Proposition 9. *(Forest Size IV) There exist unique values for $N^*(\bar{\pi})$ and $N^*(\underline{\pi})$ such that Foreign will never make acceptable offers in states in which $N > N^*(\pi_t)$ and will always make acceptable offers in states with $N \leq N^*(\pi_t)$ for the appropriate value of π_t.*

(i) $N^*(\bar{\pi})$ is given by the largest value of N for which the following equation holds

$$\frac{u(N) - u(N-1)}{1-\delta} \geq \bar{\pi} \tag{24}$$

or by 0 if the above is not satisfied for any positive value of N.

(ii) Let $N' := N - N^*(\bar{\pi})$. Then $N^*(\underline{\pi})$ is given by the largest value of N for which the following equation holds

$$\begin{aligned} u(N) - \mathbb{1}_{\{N'\geq 2\}} u(N-1)\frac{1-\delta}{1-\delta p} - \mathbb{1}_{\{N'\geq 3\}}(1-\delta)\sum_{i=1}^{N'-2}\frac{\delta^i(1-p)^i}{(1-\delta p)^{i+1}}u(N-i-1) \\ \mathbb{1}_{\{N^*(\bar{\pi})>0\}}\frac{\delta^{N'(1-p^{N'-1})}}{(1-\delta p)^{N'-1}}u(N^*(\bar{\pi})) \geq \underline{\pi} - \delta^N[p\underline{\pi} + (1-p)\bar{\pi}] \\ -\mathbb{1}_{\{N\geq 2\}}(1-\delta)\sum_{i=1}^{N-1}\frac{\delta^i(1-p)^{i-1}}{(1-\delta p)^i}\underline{\pi} + \mathbb{1}_{\{N\geq 2\}}(1-\delta)\sum_{i=1}^{N-1}\frac{\delta^N(1-p)^{i-1}}{(1-\delta p)^i}[\underline{\pi} + (1-p)\bar{\pi}] \\ -\mathbb{1}_{\{N^*(\bar{\pi})>0\}}\frac{\delta^{N'}(1-p)^{N'-1}}{(1-\delta p)^{N'-1}}[p\underline{\pi} + (1-p)\bar{\pi}][1-\delta^{N^*(\bar{\pi})}] \end{aligned} \tag{25}$$

or by $N^*(\bar{\pi})$ if it is the above does not hold for any $N > N^*(\bar{\pi})$.

[11] Note that $N^*(\underline{\pi})$ is only a useful analytical concept if 19 holds. Otherwise lemma 3 states that Home never consumes when $\pi_t = \underline{\pi}$ so that $N^*(\underline{\pi}) = N$ for all N.

The intuition for the proposition is similar to that of Proposition 4, with some minor adjustments to account for the changes in the value of π. Part (i) states that Foreign will conserve the forest in periods when $\pi_t = \underline{\pi}$ whenever the benefit of preserving an additional tree in perpetuity, represented by the left-hand side of equation 24, is equal to the marginal cost of a conserving an additional tree ($\bar{\pi}$). As in 4, the fact that $u(\cdot)$ is concave in N, implies that it becomes more difficult to satisfy this condition as the forest gets larger and ensures the existence of a unique $N^*(\bar{\pi})$. Moreover, comparing equation 24 to equation 10 from Proposition 4, it is easy to see that the inequality in equation 24 is more restrictive. This implies that price volatility leads to smaller forests, when compared to a constant price with the average $\pi = p\underline{\pi} + (1-p)\bar{\pi}$; conservation will not enter a steady state until Foreign is willing to conserve an additional tree in perpetuity at the larger marginal cost $\bar{\pi}$.

Part (ii) sets out the conditions required for Home to conserve the forest when $\pi_t = \underline{\pi}$. The left-hand side of equation 25 represents the marginal benefits to Foreign of conserving the Forest in the current period. Since conservation no longer leads to a steady state, Home now accounts for the fact that, even though it may conserve in the current period, the forest will continue to be consumed whenever $\pi_t = \bar{\pi}$. Similarly, the right-hand side of equation 25 represents the marginal cost of conservation, accounting for the fact that Foreign will make smaller payments as the forest is gradually consumed until a steady state is reached. The proof of the Proposition shows that despite these complications, that it becomes more difficult to satisfy equation 25 as N' grows larger, thereby implying that it remains more difficult to conserve larger forests and that $N^*(\underline{\pi})$ is unique.

Comparing equation 25 to equation 10, we can see that price volatility reduces the cost of conservation when the price is low. This occurs both because $\pi_t = \underline{\pi} < \pi$, but also because Home has a weaker incentive to consume when she expects that delaying consumption might increase the value of π. For example, consider equation 25 when we set $N' = 1$,

$$u(N(^*(\bar{\pi}) + 1) - u(N(^*(\bar{\pi})) > \underline{\pi}(1 - \delta p) - \delta(1-p)\bar{\pi}$$

It is straightforward to verify that the right-hand side of the equation is smaller than $(1-\delta)\underline{\pi}$ thereby implying that the potential of a potential increase in the value of π in future periods reduces the price of conservation in the current period.

8 Conclusion

Payment for ecosystem service programs like REDD+ present two competing goals: conservation of critical ecological resources and the desire to adequately compensate the resource owners - who are often from the Global South - for forgoing the consumption of those resources crucial to their development. In this article, we maintain that when designing agreements, there exists a trade-off between these two goals. We show that when the conservationist can offer binding agreements, the resource owner receives none of the surplus generated by conservation. Moreover, it is possible this arrangement maximizes the size of the forest that is conserved. However, when leakage is present, the forest size will be maximized by granting the resource owner the power to dictate the terms of agreements and both countries can receive a share of the bargaining surplus. Unfortunately, under these circumstances we have shown that conservation may not be a steady state, such that the forest will shrink even after conservation is successfully implemented at one time. Though, it should be noted that this may always be a feature of conservation agreements if the market price of forests fluctuates.

Much work remains to be done to understand the strategic considerations underlying conservation agreements. First, this article considered a complete information case. If states had private valuations for their utility for conservation, agreements might be more difficult to negotiate. A particularly interesting case would be that in which the resource owner also has private information regarding their valuation for conservation relative to consumption of the forest. Second, there exists empirical work that attempts to quantify the costs required to prevent deforestation. Most of this research is conducted at the sub-national level, where for example, economists have tried to determine the policies necessary to deter private actors from engaging in deforestation (Souza-Rodrigues 2019) or quantify the social cost of deforestation (Assunção et al. 2023). Future work should consider the costs of preventing deforestation for international actors while accounting for bargaining dynamics. Finally, conservation agreements need not only be reached for forests. However, other environmental resources might have different characteristics that make strategic dynamics more complex. For example, while deforestation uses CO2 emissions as a standardized measure to quantify the value of conservation agreements, it is not immediately obvious how one would measure the value of conserving endangered species.

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Online Supplemental Appendix

Conservation for Sale: International Bargaining over Payment for Ecosystem Services

Contents

1 Proofs of Results in the Main Text

1.1 Proof of Proposition 1

We will show that the set of countries' payoffs for being in any given state is a singleton and independent of history. To prove this, we show that any postulated strategy that does not guarantee this payoff to both players cannot survive the one-shot deviation principle (Fudenberg and Tirole 1991). The general proof strategy is proof by induction.

The proof will require that we consider history-dependent strategies. Let h_t denote the history at time t and let $s_F(N_t|h_t) = F(x|N_t|h_t)$ denote the distribution of Foreign's possibly history-dependent offer. Finally, let $V_H(1, s_F(N_t|h_t)|N, h_{t+1})$ denote Home's continuation value in case of acceptance and $V_H(0, s_F(N_t|h_t)|N_T - 1, h'_{t+1})$ denote Home's continuation value in case of rejection. We begin by showing that then $N = 1$ Home's continuation value in any SPE is π.

Lemma A. 1. *If $N_t = 1$, then Home's continuation value is any SPE is π.*

Let $\underline{V}_H(1|1, h_t)$ be Home's minimum continuation value when she accepts the offer $x_t(1, h_t)$. Since Home can always consume, she can guarantee herself π, so $\underline{V}_H(1|1, h_t) \geq \pi$.

Let $\bar{V}_H(1|1, h_t)$ be Home's maximal continuation value at an arbitrary history h_t. Suppose $\bar{V}_H(1|1, h_t) > \pi$. This inequality can be true if and only if Foreign's expected payments to Home exceed π. Assuming that Home's strategy at history h_t has it choosing to chop down the tree in k periods, the inequality holds if

$$\sum_{\tau=t}^{t+k} \delta^{t-\tau} E[x_\tau(1|1, h_\tau)] > \pi - \delta^k \pi \tag{A. 1}$$

Foreign's expected utility of this strategy is

$$\sum_{\tau=t}^{t+k} \delta^{t-\tau}(u_F(1) - E[x_\tau(1|1, h_\tau)]).$$

However, in period t, Foreign can reduce the payments. That is, since the sum of these payments is strictly greater than π, there exists a x'_t such that $x'_t < E[x_t]$ and satisfies

$$\sum_{\tau=t}^{t+k} \mathbb{1}_{\tau \neq t} \delta^{t-\tau} E[x_\tau(1|1, h_\tau)] + x'_t > \pi - \delta^k \pi \tag{A. 2}$$

Foreign is strictly better off making this smaller payment and Home will continue to prefer to play their strategy and wait at least until k before chopping. By the one-shot deviation principle, there is no SPE where $\bar{V}_H(1|1, h_{t+1}) > \pi$ and $\bar{V}_H(1|1, h_t) \leq \pi$. But since $\underline{V}_H(1|1, h_t) \leq \bar{V}_H(1|1, h_t)$, we have $V_H(1|1, h_t) = \pi$. ■

Lemma A. 2. *If $N_t = N$, then Home's continuation value is any SPE is $\sum_{i=1}^{N} \pi \delta^{i-1}$.*

Let $V_H(N-1) = \sum_{i=1}^{N-1} \pi\delta^{n-1}$. We want to show that $V_H(N) = \pi + \delta V(N-1) = \sum_{i=1}^{n} \pi\delta^{N}$ and will follow a similar proof strategy as Lemma A. 1

Let $\underline{V}_H(1|N, h_t)$ be Home's minimum continuation value when she accepts the offer $x_t(N, h_t)$. Given the inductive hypothesis, $\underline{V}_H(1|N, h_t) \geq \pi + \delta V_H(N-1)$. If this were not the case, then Home could deviate from their strategy, consume, and achieve this continuation value.

Let $\bar{V}_H(1|N, h_t)$ be Home's maximal continuation value at an arbitrary history h_t after accepting offer $x_t(N, h_t)$. Given the inductive hypothesis, $\bar{V}_H(1|N, h_t) \leq \pi + \delta V_H(N-1)$. Suppose not. That is, suppose that $\bar{V}_H(N, h_t) > \pi + \delta V_H(N-1)$. Once again, this inequality is true if and only if Foreign expected payments to Home exceed π. Assuming that Home's strategy at history h_t has it choosing to chop down the tree in k periods, the inequality holds if

$$\sum_{\tau=t}^{t+k} \delta^{\tau-t} E[x_\tau(1|N_t, h_\tau)] > [1-\delta^k][\pi + V_H(N-1)] \tag{A. 3}$$

Foreign's expected utility of this strategy is

$$\sum_{\tau=t}^{t+k} \delta^{t-\tau}(u_F(N) - E[x_\tau(1|N_t, h_\tau)]) - V_F(N)\delta^t[1-\delta^k]$$

However, in period t, Foreign can reduce the payments. That is, since the sum of these payments is strictly greater than $\pi + \delta V(N-1)$, there exists an x'_t such that $x'_t < E[x_t]$ and satisfies

$$\sum_{\tau=t}^{t+k} \delta^{\tau-t} \mathbb{1}_{\tau \neq t} E[x_\tau(1|h_\tau)] + x'_t > [1-\delta^k][\pi + V_H(N-1)] \tag{A. 4}$$

Foreign is strictly better off making this smaller payment and Home will continue to prefer to play their strategy and wait at least until k before chopping. It follows that by the one-shot deviation principle $\bar{V}_H(1|N, h_t) \leq \pi + \delta V(N-1)$. But since $\underline{V}_H(1|N, h_t) \leq \pi + \delta V_H(N-1)$, we have $V_H(1|N, h_t) = \pi + \delta V(N-1)$=$\sum_{i=1}^{N} \pi\delta^{i-1}$.

Since $V_H(1|1, h_t) = \pi$, by induction, if $N_t = N$, then Home's continuation value is any SPE is $\sum_{i=1}^{N} \pi\delta^{i-1}$ ■

Together, these lemmas demonstrate that Home's utility will be given by equation 6. To show that Foreign's utility will be given by 5 it is only necessary to note that if the countries enter into an agreement, Foreign's utility must be that which they obtain from conservation minus any payments that they make. The lemmas require that these payments be given by 6. ■

1.2 Proof of Proposition 2

Lemma A. 3. *Any SPE equilibrium offer $x(1|h_t) = \pi(1-\delta)$.*

Home will accept an offer $x(1|h_t)$ if and only if $x(1|h_t) + \delta V(1|h_t) \geq \pi$ otherwise Home will prefer to reject the offer and chop. Per the previous lemma, $V(1|h_t) = \pi$, implying that $x(1|h_t) \geq \pi(1-\delta)$. Following a similar logic to that in the previous lemma, this inequality must hold strictly. Suppose not, that is, suppose that there were an offer $x(1|h_t) + \delta\pi > \pi$. Then there would exist an $x(1|h_t)' < x(1|h_t)$ for which $x(1|h_t)' + \delta\pi > \pi$. Foreign will strictly prefer to deviate this lower offer and Home will still be willing to accept it. It follows that any offer $x(1|h_t) > \pi(1-\delta)$ cannot survive the one-shot deviation principle.

Following a similar logic, Home strictly prefers to reject and consume than reject and conserve. This is because Home would only take a current period payoff of 0 instead of π if it expected Foreign to play a history-dependent strategy to offset the loss from consumption with an offer greater than $x(1|h_t) = \pi(1-\delta)$ in future periods. However, for the reasons stated above, such an offer does not survive the one-shot deviation principle. This suffices to prove the claim.■

Lemma A. 4. *Any SPE equilibrium offer $x(N) = V(N)(1-\delta)$.*

For any state N, Home will accept an offer $x(N|h_t)$ if and only if $x(N) + \delta V(N) \geq \pi + \delta V(N-1)$ otherwise Home will prefer to reject the offer and chop. Following a similar logic to that in the previous lemma, this inequality must hold strictly. Suppose not, that is, suppose that there were an offer $x(N) + \delta V(N) > \pi + \delta V(N-1)$. Then there would exist an $x(N)' < x(N)$ for which $x(N)' + \delta V(N) > \pi + V(N-1)$. Foreign will strictly prefer to deviate this lower offer and Home will still be willing to accept it. It follows that any offer $x(1|h_t) > \pi(1-\delta)$ cannot survive the one-shot deviation principle. This suffices to show that the equality holds strictly.

To show that $x(N) = V(N)(1-\delta)$, just note that $\pi + \delta V(N-1) = V(N)$, so that rearranging $x(N) + \delta V(N) = \pi + \delta V(N-1)$ we obtain the desired expression. Finally, note that as in the previous lemma, Home strictly prefers to reject and consume than reject and conserve, since the history-dependent offers necessary to sustain such a startegy do not survive the one-shot deviation principle. ■

1.3 Proof of Proposition 3

Follows directly from the proof of Proposition 2 ■

1.4 Proof of Proposition 4

Following Proposition 3, in any given state the minimal acceptable offer that Foreign can make is given by $x(N) = (1-\delta)\sum_{i=1}^{N} \pi\delta^{i-1}$. In any given state with $N > 1$ trees, Foreign can always decide to either preserve the Forest at the current state for a discounted expected utility of $\frac{u(N)-x(N)}{1-\delta}$, or they can choose to allow Home to chop down a section of forest and begin to pay Home to conserve in the following period for a utility of

$$u(N-1) + \delta\frac{u(N) - x(N)}{1-\delta}$$

Comparing these two quantities, Foreign will choose to conserve in the current state as opposed to delaying for one more period before doing so whenever it is the case that

$$\frac{u(N) - x(N)}{1-\delta} > u(N-1) + \delta\frac{u(N-1) - x(N-1)}{1-\delta}$$

Substituting in for $x(N)$ and $x(N-1)$ we have

$$\frac{u(N)}{1-\delta} - \sum_{i=1}^{N} \pi\delta^{i-1} > u(N-1) + \delta\frac{u(N-1)}{1-\delta} - \delta\sum_{i=1}^{N-1} \pi\delta^{i-1}$$

Rearranging, we can produce the expression in equation 10.

Inspecting the result, we find that while the right-hand side of equation 10 is a constant, the left-hand side of the equation is decreasing in N. This implies that if the equation is satisfied for any given N it must be satisfied for any number smaller than N. Similarly, if the equation does not hold for a given N, it cannot hold for any number larger than N. It follows that the largest N for which equation 10 is satisfied must be unique and have the properties described in Proposition 4. ■

1.5 Proof of Proposition 5

The proof is once again by induction and follows a similar structure to that of Propositions 1 and 2. However, there are some subtle differences to account for the absence of binding agreements.

Lemma A. 5. *Let $N_t = 1$. There is a unique SPE in which Foreign makes positive offers. In this case Foreign plays a Markov strategy and always offers*

$$x^{**}(1) = \frac{\pi(1-\delta)}{\delta} \tag{A. 5}$$

and Home accepts and conserves. As a result, Foreign has a unique payoff from agreements

$$V_F(1) = \frac{u(1)}{1-\delta} - \frac{\pi}{\delta} \tag{A. 6}$$

and Home has the unique payoff

$$V_H(1) = \frac{\pi}{\delta} \tag{A. 7}$$

Suppose $N_t = 1$ and at history h_t Foreign makes a sequence of proposals $\{x_i\}_{i=t}^{\infty}$. Since consuming by Home ends the game, these sequences represent complete contingent plans at each history, should the game continue for any length.

By subgame perfection, Home will accept any stream of payments that sum to more than the current value of consumption. This implies that when $N_t = 1$, Home will accept any offer x_{t+k} at history h_{t+k} and conserve if and only if

$$\sum_{\tau=t+k}^{\infty} \delta^{\tau} E[x_{\tau}] \geq \pi + x_{t+k}. \tag{A. 8}$$

Given this requirement, it is easy to see that Foreign playing $\{x_i\}_{i=t}^{\infty}$=$\{\frac{1-\delta}{\delta}\pi\}_{i=t}^{\infty}$ and Home always accepting and not consuming is a SPE with $N_t = 1$ as this offer causes equation A. 8 to hold with equality, thereby maximizing Foreign's utility at each t.

All that remains to show is that there are no SPE sequences of pure or mixed strategies $\{x_i\}_{i=t}^{\infty}$ from history h_t such that $x_{t+j} \neq x^{**}(1)$. To see this, suppose that Foreign's offer strategy (possibly mixed) that satisfies the condition of equation (A. 8) and that for some $j > t$, Foreign chooses a $\hat{x}_j > x^{**}(1)$. For Foreign not to be strictly worse off than offering x^{**} at every history, it must be the case that in the future Foreign makes an offer $\hat{E}[x_k] < x^{**}(1)$. However, at h_j, Home's SPE strategy has it take the payment and consume because the continuation value must be less than required by equation A. 8. This makes Foreign strictly worse off than offering $x^{**}(1)$ at j.

Alternatively, suppose that Foreign's offer strategy satisfies the condition of equation (A. 8), but for some $j > t$, Foreign chooses a $\hat{x}_j < x^{**}(1)$. To ensure that this strategy satisfies equation (A. 8), Foreign must now play a strategy such that at time k $E[\hat{x}_k] > x^{**}(1)$ at a time $k > j$. That is, to offset the lower offer in an earlier period, a larger offer must be made later. However, such a strategy does not survive the one-shot deviation principle as Foreign would strictly prefer to deviate to offering $x^{**}(1)$ at time k, and Home must still accept, making it a profitable deviation. ■

Lemma A. 6. *Let $N_t = N$. There is a unique SPE in which Foreign makes positive offers. In this case, Foreign plays a Markov strategy and always offers*

$$x^{**}(N) = \frac{N\pi(1-\delta)}{\delta} \tag{A. 9}$$

and Home accepts and conserves. As a result, Foreign has a unique payoff from agreements

$$V_F(N) = \frac{u(N)}{1-\delta} - \frac{N\pi}{\delta}. \tag{A. 10}$$

and Home has the unique payoff

$$V_H(N) = \frac{N\pi}{\delta}. \tag{A. 11}$$

The proof is by induction. We therefore assume that in the state with $N-1$ trees, Foreign offers makes an offer of $x^{**}(N-1) = \frac{(N-1)\pi(1-\delta)}{\delta}$ and Home accepts and conserves. This leads to a steady state where Foreign has utility

$$V_F(N-1) = \frac{u(N)}{1-\delta} - \frac{(N-1)\pi}{\delta}.$$

and Home has utility

$$V_H(N-1) = \frac{(N-1)\pi}{\delta}.$$

We need to show that Foreign's strategy is given by equation 11, and that Foreign's and Home's utility are given by equations 12 and 13 respectively.

Once again, subgame perfection requires that Home accept any stream of payments that sum to more than the current value of consumption and its continuation value. For

$N_t = N$, Home will accept and any offer x_{t+k} at history h_{t+k} and will conserve if and only if

$$\sum_{\tau=t+k}^{\infty} \delta^{\tau} E[x_{\tau}] \geq \pi + x_{t+k} + V_H(N-1) \tag{A. 12}$$

where the value of $V_H(N-1)$ is given by the inductive hypothesis.

It is again simple to check that Foreign playing $x^{**}(N)$ as defined in equation 11 and Home always accepting and conserving constitute an SPE as this offer causes equation A. 12 to hold with equality thereby maximizing Foreign's utility at each t.

Again, it only remains to show that there are no SPE sequences of pure or mixed strategies $\{x_i\}_{i=t}^{\infty}$ from history h_t such that $x_{t+j} \neq x^{**}(N)$. As before, to see this suppose that Foreign is playing an offer strategy (possibly mixed) that satisfies equation A. 12 and that for some $j > t$, Foreign chooses a $\hat{x}_j > x^{**}(N)$. For Foreign not to be strictly worse off than offering x^{**} at every history, it must be the case that in the future Foreign makes an offer $E[\hat{x}_k] < x^{**}(N)$ at a time $k > j$. However, such a strategy does not survive the one-shot deviation because at time k, Home has a strictly profitable deviation to accepting the offer made at time $k-1$ and then consuming the tree. This is easily shown by observing that for Home not to consume it must be the case that Foreign's offer satisfies

$$x_t + \delta[E[x_{t+1}] + \pi + V_H(N-1)] \geq x_t + \pi + \delta V_H(N-1)$$

Recall that the right-hand side of this equation is equal to the stream of all Foreign's future payments when equation A. 12 holds. Given the inductive hypothesis simplifies down to

$$E[x_{t+1}] \geq \frac{N\pi(1-\delta)}{\delta}$$

This is sufficient to show that Foreign can make no offer smaller than $x^{**}(N)$ in any period.

To see that Foreign never makes a larger offer, it is sufficient to note that Home's stream of payments must satisfy equation A. 12 with equality. This is not possible with any payment larger than $x^{**}(N)$ if $x^{**}(N)$ is the minimum payment Foreign makes. ■

Together these two lemmas suffice to prove the proposition. ■

1.6 Proof of Corollary 3

To see that Δ is positive for all N and $\delta < 1$ it is simply sufficient to note that $\Delta(1, \delta) = \frac{\pi}{\delta} - \pi > 0$ and observe that the utility in equation 13 at a rate of $\frac{pi}{\delta}$ as N increases while utility in equation 6 increases at the slower rate $\delta\pi$. It is straightforward to show that $\Delta(N, 1) = 0$. ■

1.7 Proof of Proposition 6

The proof of this Proposition follows identical steps to that in Proposition 4. ■

1.8 Proof of Proposition 7

First, observe that if the $\bar{Q}(t) < N$ then the game is equivalent to that in which Foreign makes offers without commitment. This implies that any SPE that has Foreign make positive offers and Home conserve must feature both countries playing the strategies described in Proposition 5. This suffices to prove the claim when $\bar{Q}(t) < N$.

When $\bar{Q}(t) = N_t$ for all t, to show that Foreign always sets $Q = N$, we must eliminate any candidate equilibria in which Foreign plays sets $Q < N$ with positive probability. We conduct a proof by induction starting with the case where $N = 1$. The first thing to note, is that if Foreign ever makes an offer with $q = 0$, Lemma A. 5 demonstrated that Home will consume in period t unless it expects $x_{t+1} > x^{**}(1)$ as defined. Similarly, Lemma A. 1 demonstrated that Home will consume in any given period if Foreign's stream of transfers sums to less than π. Foreign will always meet this restrictions with equality, implying that the upper bound of Home's payoff is $\frac{\pi}{\delta}$.

Next, suppose that Foreign offered $q = 1$ and $x^*(1)$. If Home rejects the offer and preserves, the most it could hope for in the following period is a utility of $\frac{\pi}{\delta}$ which means that it is indifferent between a stream of $x^*(1)$ and rejection if that is offered by the highest possible payoff. Similarly, lemma A. 1 demonstrates that Home is indifferent between consumption and accepting the offer and so cannot consume. It follows that Home has no profitable deviation to accepting $x^*(1)$. Given that this is the case, Foreign clearly prefers to set $q = 1$ so that the stream of payments required to prevent Home from consuming is only equal to π rather than $\frac{\pi}{\delta}$.

Next, consider the case where $N_t = N$. We impose the inductive hypothesis that in the state with $N - 1$ trees, Foreign sets $q = N - 1$ and offers $x^*(N)$ and Home accepts, so that Foreign and Home's utility are each given by equations 5 and 6 respectively. In this case, if Foreign makes an offer with $q = 0$, then Home will conserve if and only if

$$\sum_{\tau=t+k}^{\infty} \delta^{\tau} E[x_t] \geq \pi + x_{t+k} + \delta V(N-1) +$$

which we know must hold with equality to maximize Foreign's utility. Similarly following the logic of lemmas A. 5 and A. 6 Home will delay consuming whenever

$$x_{t+k} + \delta[x_{t+k+1} + \pi + \delta V(N-1)] \geq x_{t+k} + \pi + \delta V(N-1)$$

which implies that Foreign will only ever conserve if Foreign plays a strategy where

$$x_{t_k} = \frac{\pi(1-\delta)}{\delta} + (1-\delta) \sum_{i=1}^{N} \delta^{i-1} \pi$$

This implies that the maximum utility that Home could ever hope to achieve if Foreign offered $q = 0$ is

$$\frac{\pi}{\delta} + \sum_{i=1}^{N} \delta^{i-1} \pi$$

Similarly, if Home selected $q = N$, we know from A. 2 that the best utility Home could achieve is that in equation 6. Clearly the former is larger and represents the maximum utility that Home could achieve.

Following identical arguments to those in the case with 1 tree, it can be shown that Home must accept $x^*(N)$ when Foreign sets $q = 1$ and that doing so maximizes Foreign's utility.

Following the offer strategies, it is clear that Propositions 4 and 6 apply to determine the forest size. ■

1.9 Proof of Lemma 1

The following describes the derivation of a Markov equilibrium in mixed strategies with a non-strategic offer x in the admissible range. The proof is by induction.

With 1 tree, left Foreign has the following utility for accepting an offer

$$U_F(1, p, x) = \frac{u(1) - x}{1 - \delta}$$

and the following utility for rejecting an offer

$$U_F(0, p, x) = q \times 0 + (1 - q)[u(1) + \delta U_F(0, p, x)]$$

Rearranging

$$U_F(0, p, x) = \frac{(1 - q)u(1)}{1 - \delta(1 - q)}$$

Setting these two values equal to each other we find that Foreign is indifferent between accepting the offer or not whenever it is the case that Home chops down the tree with probability

$$q = \frac{x(1 - \delta)}{u(1) - x\delta}$$

If Home decides to chop following a rejection with 1 tree left, she receives a utility of π. By contrast, if she chooses not to cut, then Home's expected utility is given by

$$U_H(0, q, x) = px + (1 - p) \times 0 + \delta U_H(0, p, x)$$

Rearranging we have

$$U_H(0, p, x) = \frac{\delta p x}{1 - \delta}$$

Mixed strategies require that Home be indifferent between chopping and not. Setting the utility of the two actions equal to each other we find that Home is indifferent whenever it is the case that $\pi = \delta U_H(0, p, x)$. Substituting and rearranging, this leaves

$$p = \frac{\pi(1 - \delta)}{\delta x}$$

It is straightforward to verify that Home's utility in the state with one tree is equal to $\frac{\pi}{\delta}$, either by substituting in for p in $U_H(0, p, x)$ or by simply observing that for Home to indifferent between chopping and not with one tree left, Home's utility in that state must be given by $\frac{\pi}{\delta}$.

The range of acceptable offers goes from $[\frac{\pi(1-\delta)}{\delta}, u(1)]$. At the lower bound $p(\frac{\pi(1-\delta)}{\delta}|1) = 1$. At the upper bound, Home is demanding the entire surplus and $q(u(1)|1) = 1$.

We now proceed to extend this result to N trees. To do so we impose the inductive hypothesis that Home's utility for the state with $N-1$ trees is given by $U_H(N-1) = \frac{(N-1)\pi}{\delta}$. Once again, we will investigate the mixed strategies for an arbitrary $x(N)$ from the range of acceptable offers. With N trees, Home's expected utility when it refrains from chopping if its demand is rejected is given by

$$U_H(0|N) = p(x|N)x(N) + (1 - p(x|N)) \times 0 + \delta U_H(0|N)$$

so that rearranging we have

$$U_H(0|N) = \frac{p(x|N)x(N)}{1-\delta}$$

On the other hand, Home's utility for chopping down a tree following a rejection is given by $\pi + \delta U_H(N-1)$. By the inductive assumption, we can rewrite this equation as πN. It follows that Home will be indifferent between chopping and not whenever it is the case that

$$\pi N = \frac{p(x|N)x(N)}{1-\delta}$$

which can be rearranged into the expression in the proposition.

Once again, it is straightforward to verify that $U_H(N) = \frac{N\pi}{\delta}$ by solving for

$$U_H(N) = p(x|N)[x(N) + \delta U_H(N)] + (1 - p(x))[q(x)\pi + \delta U_H(N-1) + (1 - q(x))\delta U_H(N)]$$

Substituting $V(N) = \frac{N\pi}{\delta}$ and $p(x|N)$ confirms that $U_N(H) = \frac{N\pi}{\delta}$.

Now consider Foreign's utility with N trees. Foreign's utility for accepting an offer x in any given period is given by

$$U_F(x(N)|N) = \frac{u(N) - x(N)}{1-\delta}$$

On the other hand, Foreign's utility for rejecting an offer if Home chopes with probability $q(x|N)$ is given by

$$U_F(x(N), q(x|N)|N) = q(x|N)[u(N-1) + \delta V(N-1)] + [1 - q(x|N)][u(N) + \delta U_F(x(N), q(x|N)|N)]$$

Or rearranging

$$U_F(x(N), q(x|N)|N) = \frac{q(x|N)[u(N-1) + \delta V(N-1)] + [1 - q(x|N)]u(N)}{1 - \delta[1 - q(x|N)]}$$

So that Foreign is indifferent between chopping and not whenever it is the case that

$$\frac{u(N)-x(N)}{1-\delta}=\frac{q(x|N)[u(N-1)+\delta V(N-1)]+[1-q(x|N)]u(N)}{1-\delta[1-q(x|N)]}$$

Solving for $q(x|N)$ we find that

$$q(x|N)=\frac{x(N)(1-\delta)}{u(N)-\delta x-(1-\delta)[u(N-1)+\delta V(N-1)]}$$

The bounds for x are derived from the arguments in the main text. It is straightforward to check that $q(\frac{N\pi(1-\delta)}{\delta}|N)=1$ and $p(u(n)-(1-\delta)\sum_{i=2}^{N}u(i-1)\delta^{N-i}]|N)=1$. ■

1.10 Proof of Proposition 8

This proof is a simple construction of an off-path equilibrium strategy demonstrating that for any of the acceptable x's, Home can construct a strategy whereby Foreign will accept $x(N)$.

First, observe that an agreement is possible whenever the condition in equation 18 holds. To see this simply note that this equation is derived by comparing Foreign's utility from the optimal agreement in which Home makes the smallest demand that it is willing to make according to Lemma 1, $\underline{x}(N)=\frac{N\pi(1-\delta)}{\delta}$, to Foreign's utility from no agreement. It is clear that this is a necessary condition for an agreement to take place.

The following argument shows that the condition in equation 18 is sufficient for an agreement. Let $\underline{V}_F(\underline{x}(N))$ denote Foreign's lowest and highest possible expected utilities from accepting an offer from Home with positive probability. Following the discussion in the main text, we know that Foreign's utility from an agreement is given by 17. It follows that Foreign receives $\underline{V}_F(\underline{x}(N))$ when Home makes the largest demand in every state starting at N_H^* which leaves Foreign with none of the surplus from an agreement. Home can credibily threaten Foreign with this payoff because its expected utility in every state in which an agreement is reached with positive probability is independent of the particular $x(N)$ it demands as demonstrated in Lemma 1. We therefore postulate that Home threatens Foreign with this payoff if Foreign does not accept its demand with positive probability in state N_H^* according to strategy $q(x, N_H^*)$ as defined in Lemma 1. It follows that in state N_H^*, Home can make any offer to Foreign in the range defined in Lemma 1 and Foreign would be weakly prefer to accept it than reject and would strictly prefer to accept it for any $x(N_H^*)$ larger than the maximal offer Home can makes per the Lemma. ■

1.11 Proof of Lemma 2

The proof is by induction. First suppose that there is one tree and that Foreign never makes positive offers regardless of π_t. Home will chop if and only if

$$\underline{\pi}\geq\delta V_H(1)$$

where $V_H(1)$ is Foreign's utility at the beginning of the stage game before the realization of π_t. If Home's strategy stipulates that they do not chop during periods in which $\pi_t = \underline{\pi}$, then

$$\begin{aligned} V_H(1) &= p\delta V_H(1) + (1-p)\bar{\pi} \\ &= \frac{(1-p)\bar{\pi}}{1-\delta p} \end{aligned}$$

Substituting in for $V_H(1)$ in the inequality above, we find that Home will chop down the tree absent a payment if and only if the inequality in equation 2 holds.

To complete the proof, we now consider the case with N trees in which Foreign never makes positive offers in any state. We introduce the inductive assumption that

$$V(N-1) = \sum_{i=1}^{N-1} \delta^{i-1} \left[\frac{(1-p)}{1-\delta p} \right]^{i-1} \bar{\pi}$$

With N trees, Home will consume a tree if and only if

$$\underline{\pi} + \delta V(N-1) \geq \delta V(N)$$

where

$$\begin{aligned} V(N) &= p\delta V(N) + (1-p)[\bar{\pi} + \delta V(N-1)] \\ &= \frac{\delta(1-p)[\pi + \delta \bar{V}(N-1)]}{1-\delta p} \\ &= \sum_{i=1}^{N} \delta^{i-1} \left[\frac{(1-p)}{1-\delta p} \right]^{i-1} \bar{\pi} \end{aligned}$$

This result verifies the inductive assumption. Substituting this result and the expression for $V(N-1)$ into the inequality above, we find that Home will consume a tree when $\pi_t = \underline{\pi}$ if and only if the inequality in equation 19 holds. ■

1.12 Proof of Lemma 3

It is easier to solve first for Home's utility and then for Foreign's strategy. Therefore, we solve for parts (ii) and then (i) before proceeding to solve for part (v) and then (iv) and (iii) in that order.

(ii) The proof follows very similar steps to that of Proposition 1 and is by induction. Let $\underline{V}_h(\pi_t, 1|h_t)$ denote Home's minimum continuation value at the start of every period. Lemma 2 states that Home will consume in periods in which $\pi_t = \underline{\pi}$. Therefore, the worst Home can do is to chop down to the tree in any period, regardless of the value of π_t

$$\underline{V}_h(\pi_t, 1|h_t) = p\underline{\pi} + (1-p)\bar{\pi}$$

Following analogous arguments to those in the proof of Proposition 1, we can then show that $\bar{V}_h(\pi_t, 1|h_t)$, Home's maximal continuation valuation at any arbitrary history h_t, must also be given by

$$\bar{V}_h(\pi_t, 1|h_t) = p\underline{\pi} + (1-p)\bar{\pi}$$

so that $V_h(\pi_t, 1) = \underline{V}_h(\pi_t, 1|h_t) = \bar{V}_h(\pi_t, 1|h_t)$.

Now consider the case with N trees. We impose the inductive hypothesis that

$$V(N-1) = \sum_{i=1}^{N-1} p\underline{\pi} + (1-p)\bar{\pi}$$

We want to show that

$$V(N) = p\underline{\pi} + (1-p)\bar{\pi} + \delta V(N-1)$$

To do so, recall that lemma 2 implies that Home will never chop in a period in which $\pi_t = \underline{\pi}$. Once again, the worst Home could do in any given period t is to consume a tree regardless of the value of π, so that

$$\underline{V}(N|h_t) = p\underline{\pi} + (1-p)\bar{\pi} + \delta V(N-1)$$

Following similar steps to those in the proof of Proposition 1, it is again possible to show that $\bar{V}_H(\pi_t, N|h_t)$ is also equal to the expression in equation 21. It follows that $V_H(\pi_t, N) = \underline{V}_h(\pi_t, N|h_t) = \bar{V}_H(\pi_t, N|h_t)$. Furthermore, we can verify the inductive hypothesis by substituting for $V(N-1)$ in the expression above.

(i) For any number of trees N or value π_t, Home will refrain from consuming the tree if and only if

$$x(\pi_t, N) + \delta V_H(N) \geq \pi_t + \delta V_H(N-1)$$

substituting in for the values of $V_H(N)$ and $V_H(N-1)$ produced in part (ii) of the proof, provides the values of $x^*(\bar{\pi}, N)$ and $x^*(\underline{\pi}, N)$ as in equation 20. Following identical arguments to those in the proof of Proposition 1, the inequality must hold with equality as larger offers do not survive the one-shot deviation principle.

(v) The proof follows similar steps to that of the proof of Proposition 1 and is by induction. Let $\underline{V}_h(\pi_t, 1|t)$ denote Home's minimum continuation value at the start of every period. Lemma 2 states that Home will not chop in periods in which $\pi_t = \underline{\pi}$. Therefore, the worst Home can do is to chop at the first opportunity when $\pi_t = \bar{\pi}$. From the proof of lemma 2 we know that this implies that

$$\underline{V}_h(\pi_t, 1|h_t) = \frac{(1-p)\bar{\pi}}{1-\delta p}$$

Following analogous arguments to those in the proof of Proposition 1, we can then show that $\bar{V}_h(\pi_t, 1|h_t)$, Home's maximal continuation valuation at any arbitrary history h_t, must also be given by

$$\bar{V}_h(\pi_t, 1|h_t) = \frac{(1-p)\bar{\pi}}{1-\delta p}$$

so that $V_h(\pi_t, 1) = \underline{V}_h(\pi_t, 1|h_t) = \bar{V}_h(\pi_t, 1|h_t)$.

Now suppose that there are N trees remaining in the forest. We impose the inductive hypothesis that

$$V(N-1) = \sum_{i}^{N-1} \delta^{i-1} \left[\frac{1-p}{1-\delta p}\right]^i \bar{\pi}$$

We want to show that

$$\begin{aligned} V(N) &= \frac{(1-p)\bar{\pi}}{1-\delta p} + \delta V(N-1) \\ &= \sum_{i}^{N-1} \delta^{i-1} \left[\frac{1-p}{1-\delta p}\right]^i \bar{\pi} \end{aligned}$$

Once again, lemma 2 implies that Home will never chop in a period in which $\pi_t = \underline{\pi}$. That means that the worst that Home could do in any given period t is to consume a tree at the first opportunity at which $\pi_t = \bar{\pi}$ so that

$$\begin{aligned} \underline{V}_H(\pi_t, N|h_t) &= p\delta \underline{V}_H(\pi_t, N|h_t) + (1-p)[\bar{\pi} + \delta V(N-1)] \\ &= \frac{(1-p)\bar{\pi}}{1-\delta p} + \delta \frac{1-p\delta V(N-1)}{1-\delta p} \\ &= \sum_{i}^{N-1} \delta^{i-1} \left[\frac{1-p}{1-\delta p}\right]^i \bar{\pi} \end{aligned}$$

Following similar steps to those in the proof of Proposition 1, it is again possible to show that $\bar{V}_H(\pi_t, N|h_t)$ is also equal to the expression in equation 23. This completes this element of the proof.

(iv)

For any arbitrary number of trees, if $\pi_t = \bar{\pi}$, then Home will accept Foreign's offer and conserve if and only if

$$\begin{aligned} x^*(\bar{\pi}, N) + \delta V(N) &\geq \bar{\pi} + \delta V(N-1) \\ x^*(\bar{\pi}, N) &\geq \bar{\pi} \left[1 - \left(\frac{\delta(1-p)}{1-\delta p}\right)^N\right] \end{aligned}$$

where the second equality follows from substituting for the values of $V(N)$ and $V(N-1)$ found in the previous section. Following identical arguments to those in the proof of Proposition 1, this inequality must hold strictly as larger offers do not survive the one-shot deviation principle.

(iii) Per lemma 2 Home will not chop if Foreign sets $x(\underline{\pi}, N) = 0$. Moreover, per part (iv) of this proof, Foreign's offer is fixed at $x^*(\bar{\pi}, N)$ as defined in equation 22 if Foreign wants Home to conserve and zero otherwise. It follows that setting $x^*(\underline{\pi}, N)$ to any positive value is unnecessary to prevent Home from chopping in any state. Foreign's utility is therefore maximized by setting $x^*(\underline{\pi}, N) = 0$. ■

1.13 Proof of Proposition 9

We will prove each claim in turn.

(i) If the inequality in equation 19 holds, then Foreign will conserve in a period in which $\pi_t = \bar{\pi}$ if and only if

$$u(n) + x^*(\bar{\pi}, N) + \delta \left[\frac{u(n)}{1-\delta} - \frac{px^*(\underline{\pi}, N)}{1-\delta} - \frac{(1-p)x^*(\bar{\pi}, N)}{1-\delta} \right]$$
$$\geq u(n-1) + \delta \left[\frac{u(N-1)}{1-\delta} - \frac{px^*(\underline{\pi}, N-1)}{1-\delta} - \frac{(1-p)x^*(\bar{\pi}, N-1)}{1-\delta} \right]$$

or rearranging

$$\begin{aligned}
\frac{u(N) - u(N-1)}{1-\delta} &\geq x^*(\bar{\pi}, N) + \delta[V_H(N) - V_H(N-1)] \\
&\geq x^*(\bar{\pi}, N) + \delta[\delta^{N-1}(p\underline{\pi}) + (1-p)\bar{\pi})] \\
&\geq \bar{\pi} - \delta^N[p\underline{\pi}) + (1-p)\bar{\pi})] + \delta^N[p\underline{\pi}) + (1-p)\bar{\pi})] \\
&\geq \bar{\pi}
\end{aligned}$$

If the inequality in equation 19 does not hold, then Foreign will conserve in a period in which $\pi_t = \bar{\pi}$ if and only if

$$u(n) + x^*(\bar{\pi}, N) + \delta \left[\frac{u(n)}{1-\delta} - \frac{(1-p)x^*(\bar{\pi}, N)}{1-\delta} \right]$$
$$\geq u(n-1) + \delta \left[\frac{u(N-1)}{1-\delta} - \frac{(1-p)x^*(\bar{\pi}, N-1)}{1-\delta} \right]$$

or rearranging and substituting in for

$$\begin{aligned}
\frac{u(N) - u(N-1)}{1-\delta} \geq & x^*(\bar{\pi}, N) \left[1 + \frac{\delta(1-p)}{1-\delta} \right] - x^*(\bar{\pi}, N-1) \frac{\delta(1-p)}{1-\delta} \\
\geq & x^*(\bar{\pi}, N) + \frac{\delta(1-p)}{1-\delta} \left[(1-\delta) \sum_{i=1}^{N} \bar{\pi} \frac{\delta^{i-1}(1-p)^{i-1}}{(1-\delta p)^i} \right] \\
& - x^*(\bar{\pi}, N-1) \frac{\delta(1-p)}{1-\delta} \\
\geq & x^*(\bar{\pi}, N) + + \frac{\delta(1-p)}{1-\delta} \left[x^*(\pi, \bar{N}-1) + (1-\delta) \frac{\delta^{N-1}(1-p)^{N-1}}{(1-\delta p)^N} \right] \\
& - x^*(\bar{\pi}, N-1) \frac{\delta(1-p)}{1-\delta} \\
\geq & \bar{\pi} \left[1 - \frac{\delta^N(1-p)^N}{(1-\delta p)^N} \right] + \bar{\pi} \frac{\delta^N(1-p)^N}{(1-\delta p)^N} \\
\geq & \bar{\pi}
\end{aligned}$$

where the second step follows by substituting in for the values of $V_H(N)$ and $V_H(N-1)$ given by equation 23. The third step is a substitution for the value of $x^*(\bar{\pi}, N)$ given by equation 22.

(ii) When the number of trees exceeds $N^*(\bar{\pi})$, then the game does not enter into a steady state when Foreign conserves since Home will still consume in periods in which $\pi_t = \bar{\pi}$. We conduct a proof by induction. Recall that $N = N' + N^*(\bar{\pi})$. Let $N' = 1$. If the inequality in equation 19 holds and $N' = 1$, then $V_F(N)$ is given by

$$V_F(N) = \frac{p[u(N^*(\bar{\pi})+1) - x^*(\underline{\pi}, N^*(\bar{\pi})+1)]}{1-\delta p} + \frac{(1-p)[u(N^*(\bar{\pi})) + \delta V_F(N^*(\bar{\pi}))]}{1-\delta p}$$

Foreign will conserve in a period in which $\pi_t = \underline{\pi}$ if and only if

$$u(N^*(\bar{\pi})+1) - x^*(\underline{\pi}, N^*(\bar{\pi})+1) + \delta V_F(N^*(\bar{\pi})+1) \geq u(N^*(\bar{\pi})) + \delta V_F(N^*(\bar{\pi}))$$

substituting in for the value of $V_F(N^*(\bar{\pi})+1)$, we find that

$$u(N^*(\bar{\pi})+1) - u(N^*(\bar{\pi})) \geq \underline{\pi}(1-\delta p) - \delta(1-p)\bar{\pi}$$

We now consider the case with N' trees more than $N^*(\bar{\pi})$. We impose the inductive assumption that

$$\begin{aligned} V_F(N^*(\bar{\pi}) + N' - 1) = &\sum_{i=1}^{N'-1} \frac{\delta^{i-1}(1-p)^{i-1} p[u(N^*(\bar{\pi}) + N' - i) - x^*(\underline{\pi}, N^*(\bar{\pi} + N' - i))]}{(1-\delta p)^i} \\ &+ \sum_{i=1}^{N'-1} \frac{\delta^{i-1}(1-p)^i u(N^*(\bar{\pi}) + N' - i - 1)}{(1-\delta p)^i} + \frac{\delta^{N'-1}(1-p)^{N'-1}}{(1-\delta p)^{N'-1}} V_F(N^*(\bar{\pi})) \end{aligned}$$

whenever Foreign makes offers to preserve the forest in periods in which $\pi_t = \underline{\pi}$ in all states with less than N trees. It follows that if Foreign acts to preserve the forest in the state with N trees whenever $\pi_t = \underline{\pi}$, then $V_F(N^*(\bar{\pi}) + N')$ will be given by

$$\begin{aligned} V_F(N^*(\bar{\pi}) + N') = &\frac{p[u(N^*(\bar{\pi}) + N') - x^*(\underline{\pi}, N^*(\bar{\pi}) + N')]}{1-\delta p} \\ &+ \frac{(1-p)[u(N^*(\bar{\pi}) + N' - 1) + \delta V_F(N^*(\bar{\pi}) + N' - 1)]}{1-\delta p} \end{aligned}$$

Substituting in for $V_F(N^*(\bar{\pi}) + N' - 1)$, we can verify that

$$\begin{aligned} V_F(N^*(\bar{\pi}) + N') = &\sum_{i=1}^{+N'} \frac{\delta^{i-1}(1-p)^{i-1} p[u(N^*(\bar{\pi}) + N' - i) - x^*(\underline{\pi}, N^*(\bar{\pi} + N' - i))]}{(1-\delta p)^i} \\ &+ \sum_{i=1}^{+N'} \frac{\delta^{i-1}(1-p)^i u(N^*(\bar{\pi}) + N' - i)}{(1-\delta p)^i} + \frac{\delta^{N'-1}(1-p)^{N'}}{(1-\delta p)^{N'}} V_F(N^*(\bar{\pi})) \end{aligned}$$

and confirm the inductive hypothesis.

Foreign will choose to preserve in the state with $N^*(\bar{\pi}) + N'$ trees if and only if

$$\begin{aligned} u(N^*(\bar{\pi}) + N') - x^*(\underline{\pi}, N^*(\bar{\pi}) + N') + \delta V_F(N^*(\bar{\pi}) + N') \\ \geq u(N-1) - \delta V_F(N^*(\bar{\pi}) + N' - 1) \end{aligned}$$

Substituting in for $V_F(N^*(\bar{\pi}) + N')$ we find that

$$\begin{aligned} &u(N^*(\bar{\pi}) + N') - (1-\delta)u(N^*(\bar{\pi}) + N' - 1) \\ &\geq x^*(\underline{\pi}, N^*(\bar{\pi}) + N') + (1-\delta)\delta V_F(u(N^*(\bar{\pi}) + N' - 1) \end{aligned}$$

Note that at this point we can prove that $N^*(\underline{\pi})$ is unique. First, the left-hand side of the inequality is clearly decreasing in N because of the concavity of $u(\cdot)$. Second, on the right-hand side, $x^*(\underline{\pi}, N)$ is increasing in N as determined by equation 20. Finally, the inductive assumption requires that for any $n \in (0, N-1)$, Foreign has found it optimal to make an offer whenever $\pi_t = \underline{\pi}$ and never when $\pi_t = \bar{\pi}$ rather than delay a period before doing so. This implies that $V_F(N)$ is increasing in N. In conjunction, these arguments show that the above inequality is more difficult to satisfy as N increases. $N^*(\underline{\pi})$ is therefore given by the largest N for which the inequality holds.

To solve for the numerical expression for $N^*(\underline{\pi})$, we substitute for $V_F(u(N^*(\bar{\pi}) + N' - 1)$ and find that

$$\begin{aligned} u(N) - u(N-1)\frac{1-\delta}{1-\delta p} - (1-\delta)\sum_{i=1}^{N'-2}\frac{\delta^i(1-p)^{i-1}p}{(1-\delta p)^i}u(N-i-1) - \frac{\delta^{N'}(1-p)^{N'-1}}{(1-\delta p)^{N'-1}} \\ \geq \underline{\pi} - \delta^N[p\underline{\pi} + (1-p)\bar{\pi}] - (1-\delta)\sum_{i=1}^{N'-1}\frac{\delta^i(1-p)^{i-1}}{(1-\delta p)^i}\underline{\pi} \\ +(1-\delta)\sum_{i=1}^{N'-1}\frac{\delta^N(1-p)^{i-1}}{(1-\delta p)^i}[\underline{\pi} + (1-p)\bar{\pi}] \\ -\frac{\delta^{N'}(1-p)^{N'-1}}{(1-\delta p)^{N'-1}}[p\underline{\pi} + (1-p)\bar{\pi} - \delta^{N^*(\bar{\pi})}[p\underline{\pi} + (1-p)\bar{\pi}]] \end{aligned}$$

■

2 Robustness of Markov Strategies when Home Makes Offers

There exists a large number of history dependent sub-game perfect Nash equilibria to the game where Home makes offers. However, in this section, we demonstrate that many such equilibria are not renegotiation proof. While a subgame-perfect equilibrium only requires that players find committing to their strategies optimal ex-ante, renegotiation proofness requires that countries find their strategies, in particular their punishment strategies, jointly preferable to any other subgame (Farrell and Maskin 1989). In effect, this requires that all continuation values at any stage of the equilibrium path must be Pareto-unranked. Otherwise, if there where two or more subgames that were pareto ranked, then whenever the players found themselves in a pareto-inferior subgame they could jointly agree to switch to the superior one.

It turns out that, as in (Harstad 2016), the only equilibria that are renegotiation proof are in mixed strategies.[1] Consequently, the only subgame-perfect and renegotiation-proof equilibria are qualitatively similar to Markov equilibria. The following Proposition closely follows Harstad (2016, Proposition 6), extending his result to the N tree case and to the case with leasing of trees as opposed to outright purchase.

Proposition B. 1. *Any renegotiation proof equilibrium must have the following features:*

(i) *In responding to a demand Foreign plays the mixed strategy $p(x, N)$ as defined in Lemma 1, part (i). In response to rejection of its demands, Home plays a mixed strategy that can depend on History. Furthermore, Home's choice of x may change across periods in a manner that depends on history.*

(ii) $U_H = U_H^{**}$ *as defined in equation 13.*

(iii) *Agreements will be reached with positive probability in any state $N \leq N_H^*$.*

The basic intuition underlying this result is similar to that underlying Bard's (2016) renegotiation-proofness result: if there is ever a history in which Home strictly prefers not to chop following a rejection, then both Home and Foreign would strictly prefer to deviate and return to that history than have Home chop down the tree. Thus, for the threat of chopping to be credible, Home must always be indifferent between chopping and returning to any other history. This requires that Home's utility be given by U_H^{**}, for Foreign play a mixed strategy in response to demands, and Home to mix following a rejection. The mixture $p(x, N)$ is the only mixture that leaves Home indifferent between chopping and a making another demand in the following period. However, Home's strategy following a rejection might vary, since future demands are not fixed as they are in a Markov equilibrium.

Finally, N_H^* remains the earliest date at which an agreement for the same reasons as it did in the Markov case. Home still prefers to arrive at an agreement as early as possible and can induce Foreign to making an agreement with positive probability by threatening to only make high demands in future periods if Foreign does not respond to a $p(x, N)$. Moreover, the best and worst expected utilities that Home can present to Foreign have not changed. Foreign can still do no worse than allowing Home to simply consume the forest. Moreover, $x(N) = \frac{N\pi(1-\delta}{)}\delta$ is still the lowest demand Home can make while receiving a utility of U_H^{**}. Consequently, the state in which agreements can first be reached is unchanged even when strategies can be history dependent.

2.1 Proof of Proposition B. 1

The proof involves three steps. First, we characterize an agreement in the state with one tree. Second, we use an induction argument to generalize the characterization of the

[1] The literature distinguishes between weakly and strongly renegotiation proof equilibria. For a weakly renegotiation proof equilibrium to be strongly renegotiation proof it cannot be pareto inferior to any other (weakly) renegotiation proof equilibrium. As in Harstad (2016) all weakly renegotiation proof equilibria because Home's expected utility is invariant across all weakly renegotiation proof equilibria.

agreement to the state with N trees. Third, we demonstrate that Foreign and Home will reach agreements with positive probability for any $N < N_H^*$.

We begin by characterizing agreements in the state with one tree. The following two lemmas prove that Home's utility is fixed and that this requires that Foreign respond to demands by playing $p(x, N)$. These two lemmas are where we most closely follow Harstad (2016).

Lemma B. 7. *If $N = 1$, then any (weakly) renegotiation proof subgame perfect equilibrium requires that $U_H(1) = \frac{\pi}{\delta}$*

Let h denote the history of the game and $a = \{1, 0\}$ be a binary variable denoting whether Foreign respectively accepts or rejects Home's offer. Let $r(h, a)$ denote the history in the middle of the stage game after Foreign has responded to Home's demand but before Home has decided whether or not to consume the tree in case of rejection. Let $U_i(N, r(h, a))$ denote player i's ($i = F, H$) utility in the stage game with h and N trees remaining after Foreign has taken action a.

The proof is by induction. When $N = 1$, any subgame perfect equilibrium is only weakly renegotiation proof if $U_H(1, r(h, 0)) = \pi$ for all h. Clearly, Home's utility can never be lower than π because it can always cut the tree. Moreover, following (Bard 2016) $U_i(N, r(h, 0))$ can never be larger than π. Suppose not. That is, suppose that there existed a subgame in which $U_H(1, r(h, 0)) > \pi$. This utility could only exist if that subgame had at least one period in which Foreign played a strategy $\hat{\sigma}_F(x|1, h)$ accepting any offer with a higher probability than $p(x, N)$ as given by Lemma 4 part (i). Moreover, Foreign would only ever play $\hat{\sigma}_F(x|1, h)$ if Home were to chop with positive probability in a history following a rejection. If Home is chopping the tree with positive probability, then this implies that $U_i(1, r(h', 0)) = \pi$ for some h'. It follows that if there exist any period in which $U_H(1, r(h, 0)) > \pi$, there must follow a period in which $U_H(1, h) > \frac{\pi}{\delta}$ and $U_H(1, r(h, 0)) = \pi$ with Home chopping with positive probability following a rejection.

Let h'' denote a subgame in which $U_H(1, h'') > \frac{\pi}{\delta}$, thereby requiring that Foreign play $\hat{\sigma}^*$ either in that period or a following one. Let h' denote the history at a subgame for which $U_H(1, r(h', 0)) = \pi$ and Home chops with positive probability following rejection. Then at history $r(h', 0)$, Home would strictly prefer to renegotiate and revert to h'' than play their prescribed strategy. Similarly, Foreign would also strictly prefer to return to h''. This is because for their original strategy to have been subgame perfect, Foreign must have at least weakly preferred playing $\hat{\sigma}^*$ to allowing Home to consume the forest outright. It follows that Foreign would strictly prefer to return to h'' and receive the utility for conservation of the tree at history h' without payment. This suffices to show that Home's utility must be fixed at $U_H(1, h) = \frac{\pi}{\delta}$ and $U_H(1, r(h, 0)) = \pi$ for any possible history h. ■

Lemma B. 8. *Foreign responds to any offer x by playing $p(x, N)$*

Following Lemma 1 part (i), we know that if Home's continuation value is fixed at π at any history $r(h, 0) = \pi$, then subgame in which Foreign accepts a demand with positive probability must have Foreign accepting any demand with probability $p(x, 1)$. ■

The following lemmas characterize the range of possible demands, the range of Foreign's possible utilities, and the Home's possible response to those demands. These depart slightly from Harstad (2016) because we consider renegotiation proof equilibria for the rental of the tree rather than its outright sale.

Lemma B. 9. *The range of possible demands in the one tree case is given by*

$$\left[\frac{\pi(1-\delta)}{\delta}, \frac{u(0)}{1-\delta} - \pi\right] \tag{B. 13}$$

for any $\delta > 0$.

Following the discussion in Lemma 1, we know that the lowest possible offer is given by $\check{x} := \frac{\pi(1-\delta)}{\delta}$ since Foreign must accept this offer with probability 1 when playing $p(\check{x}, 1)$. Given this the maximal offer that Home can make is given by

$$u(1) - \hat{x}(1) + \delta\left(\frac{u(1)}{1-\delta} - \frac{\pi}{\delta}\right) = 0$$

where $\hat{x}$ represents the largest demand Home can make such that Foreign is indifferent between rejecting Home's demand and having it consume with probability 1, and accepting this demand and having Home demand $\check{x}(1)$ in perpetuity thereafter. Solving for $\hat{x}$ completes the proof.■

Lemma B. 10.

$$U_F(1, h) \in \left[0, \frac{u(1)}{1-\delta} - \frac{\pi}{\delta}\right]$$

$U_F(1|h) \geq 0$ or else Foreign could simply allow Foreign to consume the tree. Alternatively, the best Foreign could do is to have Home make the minimum demand in perpetuity. ■

Lemma B. 11. *Following a rejection Home plays a strategy*

$$q(x|h) = 1 - \frac{u(1) - x(1|h) + \delta V(1|h(1))}{u(1) + \delta V(1|h(0))}$$

Home must always ensure that Foreign is indifferent between accepting a present period offer and rejecting it so that

$$(1 - q(x(1)|h))[u(1) + \delta V(1|h(0))] = u(1) - x(1|h) + \delta V(1|h(0))$$

Solving for q completes the proof of the lemma.■

This represents a complete characterization of the 1 tree case.

We will now consider the case with N trees and impose the inductive hypothesis that $U_H(N-1) = \frac{(N-1\pi)}{\delta}$ and $U_F(N-1) = \sum_{i=2}^{N-1} u(i-1)\delta^{N-i}$. Next, observe that Home's utility must be given by equation 13. Given the inductive assumption that $U_H(N-1) = \frac{(N-1\pi)}{\delta}$, the proof of this claim is identical to that in Lemma B. 7. Similarly, the argument that Foreign's strategy is given by $p(x, N)$ as defined in Lemma 1 is identical to that in Lemma B. 8. The following lemma establishes the range of possible demands that Home can make and that will be accepted with positive probability.

Lemma B. 12. *The range of possible demands in the case with N trees is given by*

$$\left[\frac{N\pi(1-\delta)}{\delta}, \frac{u(N)}{1-\delta} - N\pi - \sum_{i=2}^{N} u(i-1)\delta^{N-i}\right]$$

for $\delta > 0$.

As in the one tree case, the minimum demand must be given by $\check{x} := \frac{N\pi(1-\delta)}{\delta}$ because this demand must be accepted with probability 1 when Foreign plays $p(\check{x}(N), N)$ as defined in Lemma 1 part (i). This implies that the maximal demand $\hat{x}(N)$ that Home can make is given by the following equation

$$u(N) - \hat{x}(N) + \delta\left[\frac{u(N)}{1-\delta} - \frac{N\pi}{\delta}\right] = \sum_{i=2}^{N} u(i-1)\delta^{N-i}$$

so that Foreign is indifferent between allowing Home to chop trees one by one until they are all consumed or making the maximum payment today followed by the minimal payment in perpetuity thereafter. Solving for $\hat{x}(N)$ completes the proof. ■

Lemma B. 13.

$$U_F(N,h) \in \left[\sum_{i=2}^{N} u(i-1)\delta^{N-i}, \frac{u(N)}{1-\delta} - \frac{N\pi}{\delta}\right]$$

From the inductive hypothesis, we know that the worst Foreign can do in the following period is $U_F(N-1) = \sum_{i=2}^{N-1} u(i-1)\delta^{N-i}$. It follows that the worst that Foreign's present period utility must satisfy $U_F(N)/gequ(N-1) + \delta\sum_{i=2}^{N-1} u(i-1)\delta^{N-i}$. Moreover, the best agreement for Foreign is one in which Home makes the smallest demand in perpetuity. ■

Lemma B. 14. *Following a rejection Home plays a strategy*

$$q(x(N,h)|N,h) = \frac{\delta V(N|h(1)) - x(N,h) - \delta V(N|h(0))}{u(N-1) + \delta V(N-1|h(0)) - u(N) + \delta V(N|h(0))}$$

Once again, the last part of the inductive proof is to identify Home's strategy which must keep Foreign indifferent between accepting a present period offer and rejecting it, so that

$$q(x(N,h)|N,h)[u(N-1) + \delta V(N-1|h(0))] + (1 - q(x(N,h)|N,h))[u(N) + \delta V(N|h(0))]$$
$$= u(N) - x(N,h) + \delta V(N|h(1))$$

Solving for $q(x(N,h)|N,h)$ completes the proof. ■

This completes our characterization of the N tree case.

Finally, all that remains to show is that N_H^* is the largest number of trees at which an agreement is reached with positive probability. The proof of this claim follows identical steps to that of Proposition 8. Once again, Home has an incentive to reach an agreement as soon as possible, Home is indifferent as to which x it selects, and Foreign has a range of possible utilities from an agreement. As soon N_H^* is reached and there is surplus to be had from an agreement, Home can make a "reasonable" demand and threaten Foreign with demands that leave it with none of the surplus if its demand is not accepted with positive probability.■

3 A Comparison of the Size of the Forest When Forest Makes Offers with Commitment and When Home Makes Offers

There are two normative considerations in the design of international conservation agreements. The first is the size of the forest, which produces large benefits when conserved. The second, is the desire to safeguard the resource rights of resource owners, who in the case of forests, are often poor. This model highlights that these two goals can be in direct tension with one another - increased payments to the resource owner often come at the expense of conservation. Moreover, our analysis suggests that, perhaps contrary to expectation, the resource owner will be able to extract transfers that exceed the market price for the resource both when it has proposal power and without it. In the latter case this is because the resource owner likely has the means to generate leakage and deprive any foreign party of the ability to propose binding contracts.

Our analysis also produces several clear takeaways about optimal institutional design for the maximal conservation of forests - namely that agreements will be reached at larger forest sizes when Home has bargaining power. Proposition 8 demonstrated that when Home has proposal power, agreements will be reached at an earlier date than when Foreign has proposal power and can only propose nonbinding agreements due to the presence of leakage or otherwise. However, when Home has proposal power, there is no guarantee that conservation will be a steady state - even if an agreement is reached, Home might make a demand that Foreign will reject with positive probability, thereby allowing the forest to shrink. This leaves the the conclusion uncertain and raisezs the question whether there might be any way to increase the size of the forest further.

Though we do not believe it likely that foreign countries will be able to generate binding conservation agreements, we will note that our model is ambiguous as to whether the forest will be larger if Home has proposal power when compared to Foreign having proposal power and the ability to compose binding agreements. The following corollary sets out the conditions required for the forest to be larger when Home has proposal power

Corollary 1. *$N_H^* \geq N^*$ if and only if*

$$\frac{u_F(N_H^*)}{1-\delta} - \sum_{i=2}^{N_H^*} u_F(i-1)\delta^{N_H^*-i} \geq \frac{N^*[u_F(N^*) - u_F(N^*-1)]}{\delta(1-\delta)} \tag{C. 14}$$

Unfortunately, the dependence on $u_F(\cdot)$ for all possible arguments smaller than N_H^* robs of the ability to make a clear conclusion.

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