# Signaling Strength with Handicaps

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**Abstract**

In the presence of incomplete information, strong states have an incentive to invest in costly signals that can differentiate them from weaker states. I argue that states can signal strength by handicapping themselves, deliberately reducing their combat effectiveness. In an ultimatum crisis bargaining model, I show that strong states can reduce the risk of war by making themselves weaker without reducing their demands. The key to this result is a comparative advantage that allows stronger types to fight more effectively with handicaps. This allows for an equilibrium where (1) stronger states adopt larger handicaps, thereby revealing their strength to the receiver, (2) larger handicaps are more likely to deter the receiver, and (3) the positive risk of war precludes weaker types from imitating handicap signals. The ability to reveal strength peacefully has important ramifications for theories of mutual optimism, war termination, and the relationship between parity and war incidence.

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Between June 25th and July 1st, 1862, the Union army suffered a series of devastating defeats during the Seven Days' Battles and was repulsed from Richmond. This setback pushed a quick end to the war out of reach and prompted Lincoln to reevaluate his policy on slavery. Initially, Lincoln had refrained from promoting emancipation fearing that it would lead to the secession of the remaining Union slave states and alienate Unionists throughout the Confederacy (Foner 2011, 163, 210-211). Now faced with the prospect of a longer and more uncertain war, Lincoln came to view emancipation and the imposition of costs on the Confederate populace as a "military necessity" (Foner 2011, 217). To that end Lincoln convened a cabinet meeting three weeks later in which he proposed to issue the Emancipation proclamation and declare as forever free any slave in Confederate-held territory. The sharp policy reversal left his cabinet speechless (Foner 2011, 219; Guelzo 2005, 134).

In the ensuing cabinet debate great importance was attributed to the response of Great Britain and France. Both sought to intervene on behalf of the Confederacy and had only been held at bay by Union threats.[1] Secretary of State Seward convinced Lincoln that issuing the Proclamation risked inviting European intervention by signaling weakness (Guelzo 2005, 136-137). Since the Proclamation would only apply to slaves in Confederate-held territory where the Union had no power to enforce it, it would look like the US would be trying to instigate a slave uprising. Seward warned Lincoln that, in light of the Union's recent defeat, the Proclamation would look like an act of desperation. Instead, he advised Lincoln to postpone until a victory could convince foreign audiences that the Union was not acting out of panic. Lincoln agreed and would not issue the Proclamation until after the Union victory at Antietam.

The existing bargaining literature provides us with no means for understanding this behavior. Stronger states can demand more at the bargaining table and are more likely to deter a rival. Therefore, taking actions that would improve the Union's chances of defeating the Confederacy should have increased its chances of thwarting intervention. Lincoln and

[1] See Jones (2010, 37-38, 41-42, 50, 58-59) for a description of these threats and the online appendix for an expanded discussion of British and French policy towards the Civil War.

Seward determined that emancipation would be perceived as an attempt to sow civil unrest in the Confederacy, which would certainly benefit the Union. However, they decided to hold off to avoid appearing vulnerable. Subsequent events suggest that Lincoln was wise to heed Seward's advice. In the Confederacy, news of the Proclamation was received ebulliently as a sign of Union weakness (Bashir 2015, 19-20). Similarly, British newspapers viewed the Proclamation cynically and lampooned it as "Lincoln's last card." Though Antietam pushed Britain away from intervention, Foreign Secretary Russell and Chancellor of the Exchequer Gladstone still responded to the news of the Emancipation Proclamation with renewed calls for action.[2] Why should a country appear stronger by refraining from actions that were expected to weaken their rivals?

In this article, I argue that Lincoln's actions constituted a handicap.[3] Originally developed in theoretical biology, handicaps are signals of strength that require that a country reduce their capacity to fight or refrain from using it to its fullest extent. Though countries who handicap themselves decrease their probability of winning a war, they can signal confidence in their ability to fight even at reduced strength. If weaker states are unwilling to incur the risks that handicapping poses, then strong states may use handicaps to distinguish themselves and communicate strength. In the civil war case, Lincoln and Seward were trying to convey their belief that the Union could defeat the Confederacy. They understood that their military setbacks had shrouded their strength in uncertainty and sought to reveal it by showing that they were willing to accept the risk of forgoing emancipation and the unrest it could cause.

To explore the trade-offs presented by handicap signaling, I incorporate them into a standard crisis bargaining game. In the model, a country with private information regarding its strength makes a take-it-or-leave-it (TIOLI) offer and can choose whether to simultaneously handicap itself. I show that handicaps can be used to signal strength as long as they impose

[2] For discussions of the British response to Antietam and the Emancipation Proclamation, see Jones (2010, 231-236); Jones (1992, 179); Foreman (2010, 315-319); McPherson (2002 142-146).

[3] Bashir (2015) was the first to recognize this particular episode as an instance of signaling of strength. However, our explanations as to the type of signal employed by the Union diverge.

differential costs on signalers with different levels of strength. Strong types need to be able to incur the risk of handicapping with relative ease. That is, a strong type should still be to a win a conflict handily despite being handicapped. On the other hand, a weak type should not be able to imitate the handicap without having its hope for victory plummet. The model demonstrates that when strong types are endowed with such a signaling advantage, then they can use it to set themselves apart. Specifically, stronger types will adopt larger handicaps, thereby enabling the receiver to infer that a larger handicap implies a stronger signaler. In turn the receiver is more likely to back down in response to larger handicaps. Though handicaps can deter, the receiver still fights with some positive probability. This risk of fighting while handicapped prevents weaker types from imitating the signal.

This article's main result shows that when the signaler can both handicap and bargain, she can always reveal her type. Neither bargaining alone nor signaling without bargaining can generate a similar result. Absent handicapping, the receiver can only form beliefs about the signaler's strength from the offers he receives. Because states have incentives to misrepresent their private information, the receiver makes use of the risk-reward trade-off to discourage bluffing. By increasing the risk of war in response to large demands, the receiver can deter weaker types from demanding more than they would under complete information. However, the risk-reward trade-off fails for the strongest types who the receiver would not want to fight under any circumstance. Absent handicap signaling, weaker types can take advantage of the receiver's reticence to fight and bluff by issuing the same demand as the strongest types. Handicapping fixes this issue, with its differential signaling costs ensuring that each type adopts a unique handicap signal while also preventing mimicry of their demand by weaker types.

These results contribute to a growing literature on costly signals of strength. Though Ramsay and Fey (2011) have shown that private information regarding strength is most detrimental to countries' ability to reach a bargain, the signaling literature has largely focused on signals of resolve (Green and Long 2020, 53). Instead, most of the work studying how states convey strength has explored information revelation through bargaining via the risk-reward trade-off (Dal Bó and Powell 2009; Fey and Ramsay 2007, 2016; Slantchev and Tarar 2011).[4] Green and Long (2020) provide a notable exception, arguing that states can signal strength by making secret military technology public. Wolton (2020) also provides an exception, studying the difficulties in signaling strength with sunk costs when the receiver can respond with a TIOLI offer. Slantchev (2010) studies a model where the assumptions required for handicap signaling are reversed and the Defender can impose a differentially *higher* cost on a strong Challenger who reveals its type. In this case strong states may prefer to keep their private information a secret in an attempt to ambush their rivals. Handicap signaling complements these works by introducing a novel class of signals for the signaling of strength that can be used to explain a host of interstate behavior.

In the next section, I describe handicap signals in greater detail. In particular, I trace their origins in theoretical biology, provide scope conditions for their use, and describe the conditions under which handicap signaling is likely. I then proceed to the model. First, I establish a baseline result that bargaining alone is insufficient for the strongest types to reveal their strength. Second, I study handicap signals in isolation and establish the conditions and strategies required for separation. I then present the paper's main result, integrating both the handicap signaling and bargaining results into a single model. Finally, I discuss the implications of handicap signaling for war-termination, power transition theory and mutual optimism.

## Handicap Signaling in the Wild

Handicap signals originated in theoretical biology to explain why certain birds prefer mates with fitness-reducing ornaments (Zahavi 1975). Though detrimental to the fitness of the bird, these ornaments signaled quality to a mate by indicating that an individual could survive despite the handicap. The canonical example of this behavior is the male peacock's

[4] That said, to the best of my knowledge this paper is the first to describe how the risk-reward trade-off can fail to generate separation for the strongest types.

long tail feathers, which are used in ostentatious mating displays, but which are costly to produce and can also make the peacock easier for predators to capture (Zahavi and Zahavi 1999, 32-33). Formalization of this behavior demonstrated that handicaps were an "honest" signal of mate quality so long as higher quality peacocks could more easily bear the burden of having a large tail (Grafen 1990). In this case, they would develop a tail that weaker peacocks would find too onerous to imitate. In turn, the female peacock could always infer the quality of a mate from the length of its tail.

Handicap signals have also been observed in predator-prey interactions that more closely mirror interstate conflict. Consider the problem faced by a predator encountering a group of heterogeneous prey (Nur and Hasson 1984). Though the predator would like to select the easiest prey to capture, she may not be able to observe the underlying fitness of each individual quarry. Seeking to deter pursuit, fit prey can signal their ability to escape by handicapping themselves. For example, gazelles "stot" by "leaping off the ground with all four legs held stiff" (FitzGibbon and Fanshawe 1988, 69). Stotting is time and energy-intensive and should reduce the chances of a successful escape. Once again, formalization of this behavior has shown that handicapping in this way is informative because it imposes differential costs on prey of different quality (Vega-Redondo and Hasson 1993). Fit prey stot with relative ease, thereby enabling them to stot without it posing too great a threat to their survival. By contrast, stotting would pose a larger risk to weaker prey should they wind up being chased. This enables fit prey to signal their ability to escape to predators and dissuade them from pursuing a chase.[5]

### Handicap Signals in International Relations

It is well known that states have incentives to misrepresent their private information. Because the uncertainty this generates may lead to war, strong types have an incentive to distinguish themselves by investing in costly signals. As in theoretical biology, handicaps are a means

[5] For a contradictory interpretation of stotting see Smith and Harper (2003, 61-63).

by which strong states can set themselves apart. In the context of international relations, a handicap is a deliberate reduction in a country's ability to fight a war. The requirements for handicap signaling are the same is in theoretical biology - handicaps must impose differential costs. When strong states incur relatively meager penalties for handicapping, then they can signal their strength without fear of imitation by weaker types. The following describe what are likely to be the most common forms of handicap signaling.

*Manipulating buffers:* Theoretical biologists have noted that quarry can signal strength by allowing a predator to approach before beginning their escape (Vega and Redondo 1993). In this case, the prey is attempting to signal its ability to outrun a predator by giving the predator a head start that would doom a weaker type. In international relations, allowing a rival a buffer can be interpreted as a sign of strength. Consider the dilemma faced by an ancient or medieval army on its way to besiege a city. Once the army had decided to move upon the city, it could choose to either keep its approach a secret or give the defenders warning. If the beseiging army chose the latter, then it signaled its ability to defeat even the most prepared defenders. For example, during the Roman conquest of Gaul, members of the Germanic Suvei tribe negotiated with Caesar over the acceptable length of delay for negotiations before battle (Caesar, 4.7-13). Caesar explicitly recognized the risk inherent in allowing the enemy to reinforce and agreed to a modest delay anyway. When this truce crumbled, Caesar was victorious and sent emissaries with demands to German tribes across the Rhine ahead of his imminent invasion. These notifications increased risks for Caesar, whose armies had not yet crossed the Rhine, but induced the tribes to surrender or flee (Caesar, 4.16-19).

*Underdeployment of military force:* A country can also signal confidence by declining to deploy troops or other military means available to it. For example, when China invaded Vietnam in 1979 with a force of 400,000, Vietnam choose not to substantially reinforce its frontline troops.[6] Though large quantities of troops were available for immediate deployment

[6]Estimates of the number of Vietnamese troops in the theater vary from an eighth to half the size of the Chinese army (O'Dowd 2007; Zhang 2015, 135-136).

from the Cambodian theater, reinforcements were limited to three divisions (O'Dowd 2007, 65). Vietnam's decision not to reinforce was effectively a handicap and is striking given the fact that Vietnam ultimately moved seven corps to the theater after the Chinese withdrawal (O'Dowd 2007, 72). This strategy proved to be particularly risky. Though Vietnamese defenses managed to impose large costs on the Chinese army, Vietnam also underestimated the number of Chinese troops and fell into a strategically precarious position.[7] This example illustrates that a state does not have to be certain about its rival's strength before handicap signaling, a scenario explored in a formal model in the online appendix.

*Upholding moral codes:* Just war theorists have long recognized that states may find it expedient to relax moral codes while fighting (Walzer 1977, 144-151). If following rules of engagement can impact military efficacy, then just behavior can become a symbol of strength. For example, Israel has adopted a practice of giving civilians advance warning of attacks in an attempt to prevent civilian loss of life (Inbar and Shamir 2014). This can be done by communicating directly with those present at an attack site or by "roof knocking," dropping small munitions in an attempt to get civilians to evacuate. Though the efficacy of the practice in preventing civilian casualties remains controversial, it provides potential enemies with a buffer, generating a risk that military assets or intended targets can be secreted away from the bombing site (UN 2009). The civil war example discussed above is another example where an attempt to follow a moral code served as a handicap. This is because the British and French had a history of bloody colonial and slave rebellions, which led them to perceive the risks of an uprising as a humanitarian matter (Jones 2010, 121-122).

*Handicap signaling by insurgent groups:* Insurgent groups often choose to commit acts of violence to signal strength (Kydd and Walter 2006; Bueno de Mesquita 2010). Observing variation in complexity of attacks, some scholars have argued that insurgent groups can

[7] Historians debate whether Vietnam's gamble paid off. Western sources emphasize that China withdrew badly beaten and without achieving its goal of getting Vietnam to withdraw from Cambodia (Zhang 2015, 116-117). Chinese sources dispute that this was ever their intention and argue that the conflict with Vietnam was successful in the long haul (Zhang 2015, 121). Vietnamese sources emphasize that the Chinese were thwarted off solely through the use of border troops (Chen 1987, 106). For more on the argument that events unfolded against Vietnam's favor, see Zhang (2015, 109-110, 131-132, 135-136).

reveal strength by organizing simultaneous attacks in different locations. Often these attacks cannot provide one another with mutual support, implying that their simultaneity serves no tactical military purpose. Explicitly referencing the logic of differential costs, Trebbi and Weese (2019) argue that such attacks are designed to signal strength by increasing the risk of exposure to the state's security apparatus. Only a group that does not fear defection from its members or detection by the state can incur this risk (Shapiro 2013). Examining patterns of violence in Afghanistan and Pakistan, Trebbi and Weese find that insurgent groups are more likely to coordinate simultaneous attacks where they are stronger. Studying suicide bombings executed by Boko Haram, Warner, Chapin, and Matfes (2019) make a similar observation, arguing that when multiple suicide bombings were organized to take place in the same location simultaneously their perpetrators were often women and children, the least committed combatants.

## The Bargaining Baseline

I begin the formal model by exploring bargaining strategies in an ultimatum game without signaling. I show that weak types will separate successfully by each issuing the unique demand they would make under complete information. However, the strongest types fail to separate and have their demands mimicked in equilibrium. This establishes a baseline result and demonstrates that strong types may benefit from signaling in an ultimatum game.

### Model Primitives

Two countries are engaged in an ultimatum game over a good of value 1. The Challenger makes a TIOLI demand $x \in [0, 1]$ of the Defender. If the Defender accepts the demand, then the Challenger receives a payoff of $x$, the Defender receives the remaining $1 - x$, and the game ends. If the Defender rejects the demand, then the two countries fight a war. War is modeled as a costly lottery where the winner receives the good (Fearon 1995). The Challenger's probability of winning a war will be determined by its strength $s_1 \in [\underline{s}_1, \bar{s}_1]$.

I assume that this is the Challenger's private information and that the Defender's beliefs are distributed according to the continuous, strictly increasing, and common knowledge cumulative distribution function $F$ with full support. The Challenger has probability of winning the war $p_1(s_1)$, while the Defender has the reciprocal probability of winning a war $p_2(s_1) = 1 - p_1(s_1)$.[8] To reflect the notion that strength contributes to victory in war, $p_1(s_1)$ is strictly increasing in $s_1$. Finally each country has a common knowledge cost of war denoted $c_i$ $(i = 1, 2)$.

In this environment, the Defender can only learn about the Challenger's type from its choice of demand. Therefore, a strategy for the Challenger is a mapping that determines a choice of demand for every type $\sigma_x : s_1 \rightarrow x \in [0, 1]$. A strategy for the Defender is a mapping from the observed demand to a probability with which to accept the demand $\psi : x \rightarrow [0, 1]$. This generates an expected utility to the Challenger of

$$x\psi(x) + (1 - \psi(x))(p_1(s_1) - c_1) \tag{1}$$

Throughout the paper I solve for Perfect Bayesian Equilibria (PBE). This requires that the Defender updates its beliefs using Bayes' Rule whenever possible and that both players maximize their expected utility in light of these beliefs. Let $G(s_1|x)$ denote the Defender's posterior beliefs. An equilibrium is therefore composed of a triple $(\sigma_x, \psi, G)$ satisfying the requirements for a PBE.

### Equilibrium Characterization

To discourage incentives to misrepresent the Defender confronts the Challenger with a risk-reward trade-off, accepting higher demands with a lower probability (Slantchev and Tarar 2011). Since stronger types of the Challenger have higher payoffs to fighting, they will be willing to incur more risk and make higher demands. This allows the weakest types of the Challenger to separate, each issuing the same unique demand that they would make under

[8] Throughout, I will use the subscript 1 to refer to the Challenger and 2 to the Defender.

complete information. However, above a certain threshold level of strength all types pool and make the maximal demand $x = 1$. The weakest of these pooling types are bluffing, relying on the Defender's hesitancy to fight the strongest types to demand more than they would under complete information.[9] Proposition 1 presents a formal characterization of this result and figure 1 illustrates the Challenger's strategy. (Proofs are in the appendix).

**Proposition 1**

*In the ultimatum bargaining game with asymmetric information:*

*(i) There exists a Perfect Bayesian Equilibrium in which the Challenger plays*

$$\sigma_x^*(s_1) \equiv \begin{cases} p_1(s_1) + c_2 \text{ if } s_1 < \rho \\ 1 \text{ if } s_1 \geq \rho \end{cases} \tag{2}$$

*where $\rho$ is given by*

$$\rho \equiv \left\{ s_1' : c_2 = \frac{\int_{s_1'}^{\bar{s}_1} f(s_1) p_2(s_1) ds_1}{1 - F(\rho)} \right\} \tag{3}$$

*The Defender responds by playing*

$$\psi^*(x) \equiv \begin{cases} e^{-\frac{x - p(\underline{s}_1) - c_2}{c_1 + c_2}} & \text{if } x < 1 \\ \frac{(c_1+c_2)e^{-\frac{p_1(\rho) - p_1(\underline{s}_1)}{c_1+c_2}}}{1 - p_1(\rho) + c_1} & \text{if } x = 1 \end{cases} \tag{4}$$

*and has beliefs*

$$G^*(s_1|x) \equiv \begin{cases} 1 \text{ if } s_1 = \sigma_x^{*-1}(x) \text{ and } x < 1 \\ \frac{f(s_1)}{1-F(\rho)} \text{ if } x = 1 \end{cases} \tag{5}$$

*(ii) $\sigma_x^*$ is the unique signaling function subject to D1.*

[9] This failure of separation is the key difference between the model presented here and previous work. Dal Bó and Powell (2009) show that complete separation is possible in a crisis bargaining game where the size of the pie is increasing with the Challenger's strength. Similarly, all types can separate in the isomorphic model where the Challenger has private information regarding the size of the pie (Reinganum and Wilde 1986; Reinganum 1988). By contrast, I show that these results do not hold when the size of the pie is fixed and there exist types that give the Defender a negative payoff for fighting.

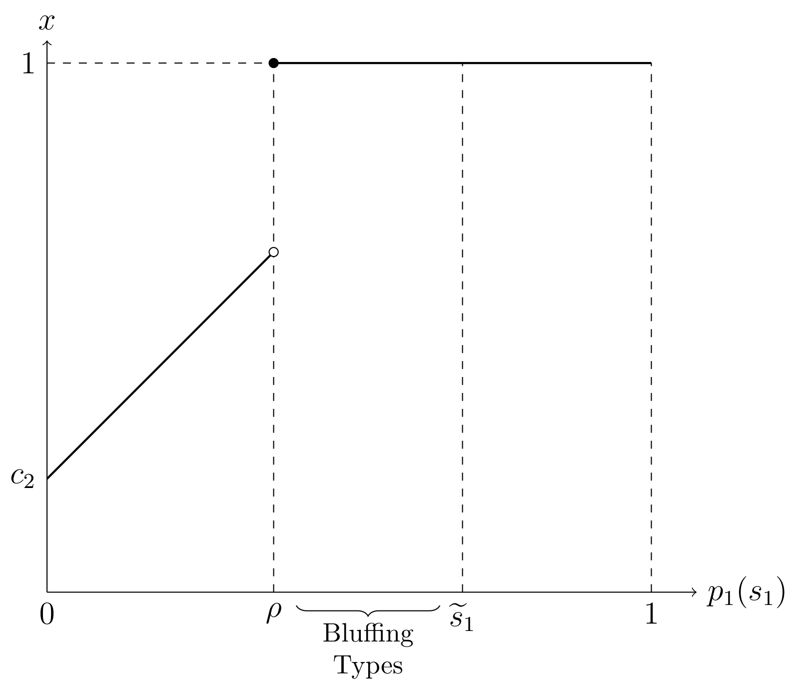


Figure 1: **Equilibrium Demands with a TIOLI Offer:** This figure plots how the Challenger's demand increases with their probability of winning. Challenger's weaker than type $\rho$ demand $x = p_2(s_1) + c_2$ so that demands increase linearly up to that point. Type $\rho$ is the weakest type to demand $x = 1$, and all types stronger than $\rho$ do the same. Types in the range $[\rho, \widetilde{s}_1)$ are called bluffing types because they are the only one whose demands do not coincide with the demands they would make if the game were one of complete information.

The equilibrium is intuitive. For the Defender to be able to sustain the risk-reward trade-off, she must be willing to mix between fighting and conceding in response to all but the lowest demands.[10] Separation is then possible if each type of the Challenger demands $x = p_1(s_1) + c_2$ such that the Defender indifferent between conceding and fighting. However, a problem arises when some types of the Challenger are too strong. Let $\widetilde{s}_1$ denote the type for whom $p_1(\widetilde{s}_1) + c_2 = 1$ so that the Defender is indifferent as to whether or not it fights type $\widetilde{s}_1$ even if makes the maximal possible demand. If $\widetilde{s}_1$ and types stronger than $\widetilde{s}_1$ are the only types to pool on the highest possible demand $x = 1$, then the Defender would strictly prefer to accept the demand than to fight. This cannot be part of an equilibrium because weaker types could then deviate and receive the good without risking war. Therefore any equilibrium must have a sufficient mass of weaker types, $[\rho, \widetilde{s}_1)$, bluff and demand $x = 1$ as well so that the Defender can be left indifferent between fighting and conceding.

Because $\sigma_x^*$ leaves the Defender indifferent in response to any demand, her strategy only needs to ensure that all types of the Challenger find $\sigma_x^*$ optimal. Each type of the Challenger will seek to increase the size of its demand until the marginal gains from doing so are equal to the marginal costs

$$-\frac{\psi'(x)}{\psi(x)} = \frac{1}{x - p_1(s_1) + c_1} \tag{6}$$

The left hand-side of this equation is the marginal cost of increasing the size of the demand and represents the reduction in the probability that the Defender backs down as $x$ increases. The right-hand side is the marginal gain from increasing the demand and represents the marginal change in the Challenger's payoff if their demand is accepted, divided by the difference in payoffs between having its demand accepted and fighting. The Defender's strategy $\psi^*$ is a solution to the ordinary differential equation in equation 6. This ensures that equation 6 will hold for each of the separating types and for type $\rho$ when it demands 1. Note that equation 6 confirms the intuition that that stronger types of the Challenger will be more

[10] A formal proof of this claim is provided in Fey and Ramsay (2016).

willing to make larger demands because they have less to lose from going to war. [11]

Finally, $\sigma_x^*$ is unique when the Defender's beliefs are subject to reasonable restrictions off the equilibrium path. If the Challenger deviates to a demand that is not part of an equilibrium, then the Defender cannot update its beliefs using Bayes' Rule. In this case, I follow Dal Bó and Powell (2009) and Fey and Ramsay (2016) and impose the D1 Criterion. Formally, this requires that the Defender believe that any off-path demand be issued by the type which can benefit from the the largest set of possible responses (Fudenberg and Tirole 1991). The proof of Proposition 1 adapts arguments from Cho and Sobel (1990) and Ramey (1996) to show that because the strongest types are more willing to risk war, D1 implies that the Defender must believe that any deviation to an off-path demand has to be made by the strongest type issuing a lower equilibrium demand. Given these beliefs, it is easy to show that any alternate equilibrium that features pooling on a demand $x < 1$ is unstable since the Defender will have to believe that the strongest pooling type is behind any deviation to an off-path demand larger than $x$ and then strictly prefer to concede. Moreover, the proof shows that D1 does not eliminate the equilibrium described in Proposition 1 as D1 would require that the Defender believe that any deviation to a demand $(p_1(\rho) + c_2, 1)$ be issued by type $\rho$ and would then strictly prefer to fight.

## Modeling Handicaps

The inability of the strongest types to separate with bargaining alone gives them an incentive to invest in costly signaling. In this section, I demonstrate that types who failed to separate with bargaining can do so with handicap signals instead. The key to achieving this result is an assumption of increasing differences, which requires that stronger types be penalized less for handicapping themselves. This allows stronger types to handicap themselves more for smaller marginal increases in the probability that the Defender concedes and discourages

[11] Formally, the Challenger's expected utility function satisfies the single-crossing property, which implies that $\sigma_x$ must be weakly increasing. This condition is easily checked by taking the cross partial of equation 1 and seeing that it is positive. For more on single-crossing, see Ashworth and Bueno de Mesquita (2006).

weaker types from imitation. To highlight the trade-offs inherent in handicap signaling I temporarily suppress bargaining and assume that the two countries are engaged in a conflict over an indivisible good.

### Model Primitives

Suppose now that the countries play an ultimatum game over an indivisible good with value 1. In place of making a demand, the Challenger will instead choose a handicap signal $h \in [0, \bar{h}]$. The Defender observes the Challenger's handicap and must then decide whether to concede the good or fight. If the Defender concedes, then it receives a payoff of 0 and the Challenger receives a payoff of 1. If the Defender chooses to fight, then both players get their wartime payoffs which are now a function of both $s_1$ and $h$. As before, the Challenger's probability of winning the war will be strictly increasing in its strength type $s_1$, which remains private information. To reflect the notion that handicaps harm a Challenger's probability of winning a war, $p_1(s_1, h)$ is strictly decreasing in $h$. Without bargaining, a strategy for the Challenger is simply a mapping from its type to a handicap choice $\sigma_h : s_1 \to h$. Following the notation in Vega-Redondo and Hasson (1993), the Defender's strategy is a mapping from the observed handicap to a probability with which to concede $\phi : h \to [0, 1]$. This implies that the Challengers utility function will be given by

$$\phi(h) + (1 - \phi(h))(p_1(s_1, h) - c_1) \tag{7}$$

We will once again be searching for a PBE, this time denoting the Defender's posterior beliefs after observing $h$ with $H(s_1|h)$.

As discussed above, handicaps can only serve as effective signals if stronger types can bear them more easily. The following assumption is a formalization of this requirement and is necessary to ensure that weak types will not want to imitate the signals sent by stronger types (Nur and Hasson 1984; Grafen 1990; Vega-Redondo and Hasson 1993).

**Assumption 1**

$$p(s_1', h') - p(s_1', h) > p(s_1, h') - p(s_1, h) \;\forall\; h' < h \in [0, \bar{h}] \textit{ if and only if } s_1 > s_1'$$

This inequality states that a weaker type $s_1'$ increasing their handicap from $h'$ to $h$ must experience a larger decrease in strength than any stronger type $s_1$. For example, the function $p_1(s_1, h) = s_1 - \frac{h}{s_1}$ satisfies this assumption as the penalty for a handicap $h$ is clearly decreasing in the Challenger's strength. The popular Tullock function often used to model military contests also satisfies this assumption when handicaps are modeled as follows $p_1(s_1, h) = \frac{s_1 - h}{s_1 - h + s_2}$ where $s_2$ denotes the Defender's constant level of strength.[12] Note that Assumption 1 is equivalent to requiring that a lottery function have a positive cross-partial $\frac{\partial^2 p_1(s_1,h)}{\partial s_1 \partial h} > 0$ (Ashworth and Bueno de Mesquita 2006).

While Assumption 1 is a necessary condition for handicaps to serve as a viable strategy, a number of additional convenience assumptions can make the problem more tractable and simplify the game tree. First, I assume that all types of the Challenger still find war profitable without a handicap so that

$$p_1(\underline{s}_1, 0) - c_1 > 0 \;\forall\; s_1 \tag{8}$$

This eliminates the need to model an initial decision by the Challenger of whether or not to demand the good. Second, I assume that the Defender will strictly prefer to stand firm if no signal is sent ($h = 0$) so that

$$\int_{\underline{s}_1}^{\bar{s}_1} p_2(s_1, 0) dF(s_1)\, ds_1 - c_2 > 0 \tag{9}$$

Third, I assume that there exists a type of the Challenger which the Defender would prefer not to fight. That is, there exists a type $\widetilde{s}_1 \in (\underline{s}_1, \bar{s}_1)$ such that $p_2(\widetilde{s}_1, 0) = c_2$. Together,

[12] I am thankful to an anonymous reviewer for suggesting this example.

these convenience assumptions imply that under complete information war with the strongest types can be avoided but will always occur under asymmetric information absent signaling.

**Equilibrium Characterization**

This setup produces a semi-separating equilibrium in which the strongest types of the Challenger handicap themselves and the weakest types do not. Specifically, signaling states will separate, each adopting a unique level of handicap with stronger types of the Challenger choosing to handicap themselves more. In turn, the Defender can infer that a larger handicap implies a stronger Challenger and is more likely to back down in response. The Defender can also conclude that the absence of a handicap indicates a Challenger that the Defender would strictly prefer to fight. The following proposition provides a complete characterization of this equilibrium and demonstrates that is unique when the Defender's off-path beliefs are once again subject to reasonable restrictions.

**Proposition 2**

*If the cross-partial on the lottery function satisfies*

$$\frac{\partial^2 p_1(s_1,h)}{\partial s_1 \partial h} > -\frac{\frac{\partial p_1(s_1,h)}{\partial s_1}\frac{\partial p_1(s_1,h)}{\partial h}}{c_1+c_2} \tag{10}$$

*then*

*(i) There exists a Perfect Bayesian Equilibrium where the Challenger plays,*

$$\sigma_h^*(s_1) \equiv \begin{cases} 0 & \text{if } p_1(s_1,0) \leq 1-c_2 \\ h : p_1(s_1,h) = 1-c_2 & \text{if } p(s_1,0) > 1-c_2 \end{cases} \tag{11}$$

*and the Defender responds by playing*

$$\phi^*(h) \equiv 1 - e^{\frac{1}{c_1+c_2}\int_0^h \frac{\partial p_1(\sigma_h^{*-1}(h),h)}{\partial h} dh} \tag{12}$$

*and has beliefs*

$$H^*(s_1|h) \equiv \begin{cases} \frac{f(s_1)}{F(\widetilde{s}_1)} & \text{if } h = 0 \\ 1 & \text{if } \sigma_h^{*-1}(h) = s_1 \text{ and } h > 0 \end{cases} \tag{13}$$

*(ii)* $\sigma_h^*$ *is the unique signaling function that allows for full separation of types* $s_1 > \widetilde{s}_1$.

*(iii)* $(\sigma_h^*, \phi^*, H^*)$ *are unique subject to the D1 Criterion.*

To understand this result, it is useful to begin by noting that any Perfect Bayesian Equilibrium must have the Defender play a mixed strategy in response to any positive handicap signal ($h > 0$). The Defender cannot play a pure strategy where it concedes in response to a given handicap signal because handicaps are only costly if the Defender fights. If a given handicap signal were to make the Defender back down with certainty, then all types would have an incentive to pool on that signal and the signal would become uninformative. The second convenience assumption in equation 9 would then imply that the Defender strictly prefers to deviate and stand firm. Alternatively, if the Defender were to stand firm with certainty, then the Challenger is better off not signaling since it would prefer to fight without a handicap. Thus, the Defender must be made indifferent between backing down and standing firm for any given handicap signal, $h \neq 0$.

The requirement that the Defender be indifferent makes the construction of an equilibrium with separation straightforward. If each type of the Challenger sends a unique signal, then they must select a signal $h$ such that $p_2(s_1, h) = c_2$.[13] Because handicaps strictly reduce the Challenger's probability of winning, types in the set $[\underline{s}_1, \widetilde{s}_1]$ cannot signal as the Defender would prefer to stand firm and fight these types even in the absence of a handicap. Together these requirements form a strategy for all types of the Challenger. This strategy is represented in Figure 2.

[13]Such an $h$ is guaranteed to exist for each type $s_1 > \widetilde{s}_1$ because $p_2(s_1, h)$ is monotonically increasing on a closed interval and so must be continuous almost everywhere (Fey and Ramsay 2011, Proposition 1).

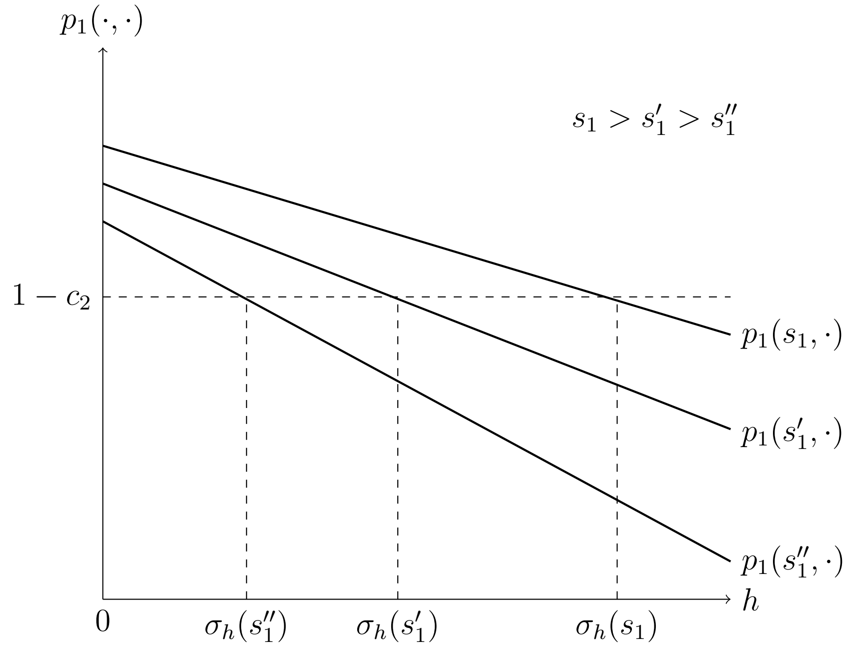


Figure 2: **The Logic of Handicap Signaling:** The y-axis captures the Challenger's probability of winning as a function of both their type $s_1$ and handicap choice $h$, which varies along the x-axis. The three sloping lines plot the changes in the probability of winning for three different types of Challenger. The Defender is indifferent between fighting and conceding the good when $p_1(\cdot,\cdot) = 1 - c_2$, represented by the horizontal dashed line. Types for whom $p_1(s_1, 0) > 1 - c_2$ handicap themselves until their probability of winning equals that amount, and their handicap choices are plotted with vertical dashed lines. The effects of Assumption 1 are captured in the figure by stronger types having smaller downward slopes in their probability of winning for handicapping themselves.

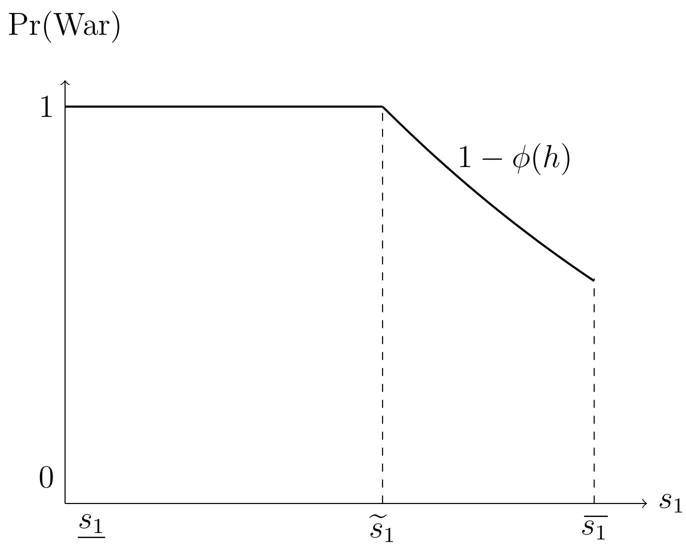


Figure 3: **The Probability of War with Signaling Only**: This figure plots the probability of war as a function of the Challenger's type. For types too weak to signal, the Defender is sure to stand firm so that the probability of war is 1. Types stronger than $\widetilde{s}_1$ handicap themselves so that the probability of war decreases by $\phi(h)$.

Given that the Defender is indifferent when the Challenger handicaps itself, it selects a strategy that ensures that $\sigma_h^*$ is incentive compatible. Once again, the Challenger will handicap itself up to the point where the marginal gain from doing so is equal to the marginal cost

$$\frac{\phi'(h)}{1-\phi(h)} = -\frac{\frac{\partial p_1(s_1,h)}{\partial h}}{1-p_1(s_1,h)+c_1} \tag{14}$$

Equation 14 is an ordinary differential equation with solution $\phi^*$. The left-hand side of this equation is the hazard rate determining the increase in probability that the Defender concedes in response to a higher handicap and represents the marginal gain from handicapping. The right-hand side of this equation represents the costs of handicapping, the decrease in the Challenger's payoff from handicapping weighted by the difference in payoffs from having the Defender concede and going to war. It follows that Assumption 1 is the driving force allowing for separation. It ensures that stronger types are willing to adopt higher handicaps in exchange for smaller marginal increases in the probability that the Defender backs down. This is captured in Figure 3 which plots the equilibrium probability of war.

Finally, Proposition 2 establishes two additional results. First, Assumption 1 and its implication that stronger types of the Challenger are better able to bear the burden of handicapping is a necessary but insufficient condition to guarantee the existence of a semi-separating equilibrium with handicap signaling. In the proof of Proposition 2, I show that equation 10 is required for the Challenger's expected utility function defined in equation 7 to be concave and subsequently for the strategies described above to constitute an equilibrium. This condition is notably more restrictive than Assumption 1 and is required because the differential costs assumption is imposed on the lottery function $p_1(\cdot, \cdot)$ instead of the utility function as a whole as is generally assumed in signaling models with differential signaling costs (Mailath 1987).[14]

Second, is that the equilibrium is unique when the Defender's off-path beliefs are required to satisfy the D1 Criterion. Per Proposition 2, part (ii) $\sigma_h^*$ is the unique strategy that allows for separation of signaling types since no other possible strategy could leave the Defender indifferent. It thus follows that any other equilibrium must have some types in the range $(\widetilde{s}_1, \bar{s}_1]$ pooling on a signal $h \geq 0$. As in the bargaining baseline, the strongest of these pooling types will seek out more risk and be those most willing to handicap themselves in exchange for a higher probability of concession. Therefore, D1 again requires that the Defender believe that any deviation to an off-path handicap $h$ must be performed by the strongest type otherwise issuing a lower handicap. As a result, any alternate equilibrium featuring pooling on a handicap $h \in (0, \sigma_h^*(\bar{s}_1)$ is unstable, since the Defender will have to believe that the strongest pooling type is behind any off-path deviation.

[14]Whether or not this condition holds depends on the functional form of $p_1(s_1, h)$. For example, if the lottery function takes the Tullock form mentioned earlier $p_1(s_1, h) = \frac{s_1 - h}{s_1 - h + s_2}$, then this condition is satisfied for all possible values $c_1$ and $c_2$. On the other hand, if the lottery function takes the form $p_1(s_1, h) = s_1 - \frac{h}{s_1}$, then handicap signaling is only possible if $1 + c_1 \geq 2\bar{s}_1$.

## Simultaneous Bargaining and Signaling

In this section I present the paper's main result. I demonstrate that it is possible for all types of the challenger to separate, either by adopting a unique demand or a unique handicap signal. This result is a natural integration of the bargaining baseline and of handicap signaling with an indivisible good. Types that separate by signaling when the good is indivisible continue to do so in the unified model, adopting identical signals in both environments. As in the bargaining baseline, these types pool on the maximal demand, effectively treating the good as indivisible. By contrast, the weakest types do not handicap themselves and each make a unique demand that leaves the Defender indifferent. Types who bluffed in the bargaining baseline, are deterred from doing so by the adoption of handicap signals by stronger types. Instead, these types revert to making the demands they would under complete information. Figures 4 and 5 illustrate the changes in the players' strategies when the Challenger can both bargain and signal.

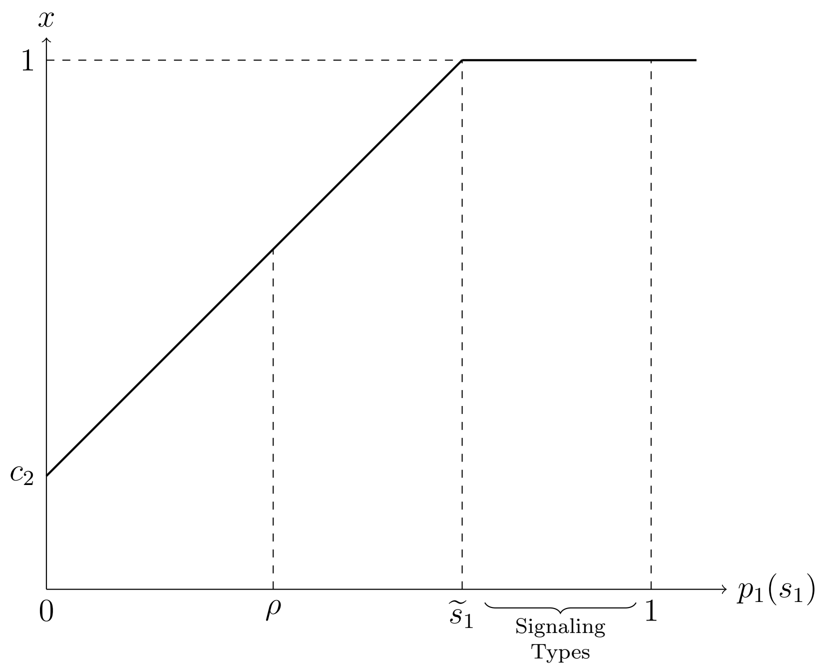


Figure 4: **Equilibrium Demands with Bargaining and Signaling:** This figure depicts the changes in the Challenger's bargaining strategy when the good is divisible and the Challenger can signal. Each type weaker than $\widetilde{s}_1$ makes a unique demand that leaves the Defender indifferent. Types stronger than $\widetilde{s}_1$ pool on demanding the whole good $x = 1$ and separate by adopting unique handicap signals. The use of handicap signals by the strongest types prevents types in the range $[\rho, \widetilde{s}_1]$ from "bluffing" as they did in the bargaining baseline.

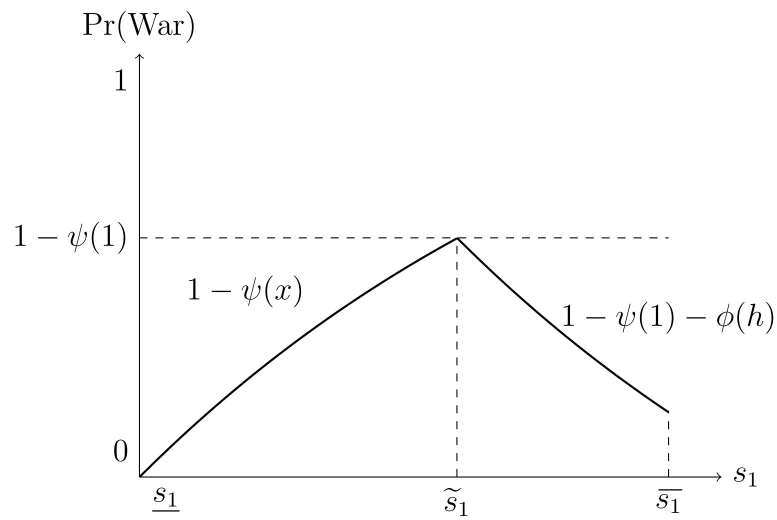


Figure 5: **The Probability of War with Bargaining and Signaling**: This figure plots the probability of war as a function of the Challenger's type. It demonstrates that the relationship is non-monotonic. Types in the range of $[\underline{s}_1, \widetilde{s}_1]$ increase their demands as they grow stronger. Higher demands lead to an increased probability of war which is designed to ensure that weaker types do not bluff. However, for types stronger than $\widetilde{s}_1$ the probability of war is decreasing. These types pool on demanding the entire good but each adopt a unique handicap signal that is increasing in type. Higher handicaps indicate a stronger Challenger and are in turn more likely to deter the Defender.

### Model Primitives

To prove this result, it is necessary to explore the interactions between bargaining and handicap signals. With simultaneous bargaining and signaling, a strategy for the Challenger is a correspondence mapping their type to a choice of demand and a handicap. Formally, $\sigma : s_1 \rightrightarrows [0, \bar{h}] \times [0, 1]$. The Defender's strategy must account for both of these endogenous variables when deciding whether to concede so that $\varphi : [0, 1] \times [0, \bar{h}] \to [0, 1]$. Anticipating the Defender's strategy, the Challenger must select a pair $(x, h)$ to maximize their expected utility, given by

$$x\varphi(x, h) + (1 - \varphi(x, h))(p_1(s_1, h) - c_1) \tag{15}$$

In turn the Defender's has the conditional posterior beliefs $J(s_1|x, h)$. Therefore, a PBE is composed of a triple $(\sigma, \varphi, J)$.

### Equilibrium Characterization

Proposition 3 provides a complete characterization of the equilibrium where the Challenger can both bargain and signal using handicaps.

### Proposition 3

*If the inequality in equation 10 holds, then*

*(i) there exists an equilibrium where the Challenger makes demands according to*

$$x^*(s_1) \equiv \begin{cases} p_1(s_1) + c_2 \ \textit{if} \ p(s_1, 0) \leq 1 - c_2 \\ 1 \ \textit{if} \ p(s_1, 0) > 1 - c_2 \end{cases} \tag{16}$$

*and signals according to equation 11. The Defender will respond by playing*

$$\varphi(x, h) = \psi(x) + \phi(h) \tag{17}$$

*where* $\psi(x) = \exp\left(-\frac{x - p(\underline{s_1}) - c_2}{c_1 + c_2}\right)$ *and* $\phi(h)$ *is identical to equation 12 and has beliefs*

$$J(s_1|x, h) \equiv \begin{cases} 1 \ \textit{if} \ s_1 = x^{*-1}(x) \ , \ x < 1 \ \textit{and,} \ h = 0 \\ 1 \ \textit{if} \ s_1 = \sigma_h^{*-1}(h) \ \textit{and} \ x = 1 \end{cases} \tag{18}$$

*(ii) the Challenger's strategy is unique subject to the D1 Criterion.*

The following describes the logic underlying the result. To begin, note that the choice of demand determines the amount the Challenger handicaps itself. The requirement that the Challenger be mixing in response to all but the lowest demand implies that if the Challenger issues a demand $x$ smaller than $p_1(s_1, 0) + c_2$, then it must be the case that the Challenger is either pooling on the demand with weaker types or that it is using handicaps signals to distinguish itself and deter the Defender. Separation requires the latter and that the Challenger handicap itself up to the Defender's point of indifference, which now occurs whenever $p_1(s_1, h) = x - c_2$. It follows that for any given choice of demand, separation is possible when the Challenger plays according to

$$h^*(x) \equiv \begin{cases} 0 \text{ if } p_1(s_1, 0) \leq x - c_2 \\ h : p_1(s_1, h) = x - c_2 \text{ if } p_1(s_1, 0) > x - c_2 \end{cases} \tag{19}$$

Because stronger types of the Challenger are better able to bear handicaps they will continue to benefit from using them whenever possible. Formally, equation 10 from Proposition 2 continues to guarantee that the Challenger's utility is concave in $h$ for any given demand. This is the sole necessary condition for separation, and is sufficient to guarantee that sufficiently strong types of the Challenger will use handicaps to separate under D1.

The requirement that the Challenger separate using handicaps when able allows us to express the Challenger's choice of handicap in terms of $x$ and transforms the Challenger's decision into a single variable maximization problem. Using equation 19, we can substitute for $p_1(s_1, h^*(x))$ into equation 15 for any Challenger that uses handicap signals. This allows us to rewrite the Challenger's utility function as follows

$$x\varphi(x, h^*(x)) + (1 - \varphi(x, h^*(x)))(x - c_2 - c_1) \tag{20}$$

This equation demonstrates that the Challenger must account for three competing effects when selecting the optimal demand. First, higher demands increase the Challenger's payoff if the demand is accepted. Second, because the Challenger needs to handicap itself less when making larger demands, increasing $x$ also increases the Challenger's payoff when its demand is rejected. Together these two effects imply that higher demands increase the Challenger's expected payoff at a linear rate. This becomes apparent when we simplify 20 into

$$x - (1 - \varphi(x, h^*(x)))(c_2 + c_1) \tag{21}$$

However, lower demands have the potential to increase the Challenger's utility by increasing the probability that a demand is accepted. While lower demands implied lower risk in the bargaining baseline, handicap signals decrease the risk of war even further. Decreasing a demand from $x$ to $x'$ increases the set of types who are sufficiently strong to handicap from $\{s_1 : p_1(s_1, 0) \geq x - c_2\}$ to $\{s_1 : p_1(s_1, 0) \geq x' - c_2\}$. Assumption 1 states that these weaker types experience larger marginal decreases in strength when they handicap. This requires that the Defender increase their rate of acceptance to lower demands at a convex rate.

Proposition 3 establishes that the Challenger's expected utility is maximized when it makes higher demands. The proof demonstrates that the first order condition of the Challenger's utility function with respect to $x$ produces a saddle point whenever the Challenger selects a demand $x$ and handicap $h$ that are both interior solutions. This implies that the Challenger's expected utility is maximized by one of two "corner" solutions. Either the Challenger maximizes the linear increase in $x$ and makes the highest demand they can while keeping the Defender indifferent. Or the Challenger reduces their demand to maximize $\varphi(x, h^*(x))$. Proposition 1 has already shown that the Challenger will prefer to increase their demand even if this increases the risk of war. Proposition 3 shows that this remains true even though the deterrent effect of handicaps will shrink as the size of the demand grows.

It is worth noting that the introduction of signaling may actually *decrease* the welfare of the strongest types. In the bargaining baseline, type $\rho$ had to be indifferent between demanding $x = 1$ and $x = p_1(\rho, 0) + c_2$. In Proposition 3, type $\rho$ strictly prefers the lower demand and the Defender has decreased the rate at which it backs down in response to $x = 1$. The strongest types demand $x = 1$ in either scenario and can handicap themselves for a higher acceptance rate in the latter. However, it is possible that the benefits from handicapping are smaller than those required to induce $\rho$ to demand $x = 1$. Though the strongest types might prefer a return to the bargaining baseline, the D1 criterion rules this out. D1 ensures that in any alternative equilibrium, off-path beliefs will be such that the strongest types of Challenger will always have an incentive to deviate, handicap themselves, and separate.

## Discussion

The previous sections demonstrated that it is possible for countries to signal strength and achieve separation using handicaps. Because the ability of countries to signal strength has not previously received much attention, there are a number of theoretical implications that follow when states can convey information during crisis bargaining that have not yet been explored. This section lays out some of these implications, which need not necessarily be unique to handicap signals.

### War Termination

A popular theory of war termination maintains that if wars are the result of incomplete information, then wars should end when states have revealed sufficient information to reach an agreement. Per this argument, wars are a costly mechanism by which states can resolve the bargaining problem posed by private information. Countries can learn from battlefield outcomes, which produce noisy but unbiased indications of a rival's strength (Wagner 2000). Additionally, countries can screen their rivals with strategic offers, threatening to continue the fight against those who refuse their peace offers during wartime (Powell 2004; Slantchev 2003). Both processes allow states to learn about their rival's strength and willingness to fight, gradually producing a convergence in beliefs.

However, handicap signals undermine the premise of this argument. In the model, I show that countries can enter a dispute with incomplete information over strength, completely overcome this uncertainty with a mixture of signaling and bargaining, and still find themselves at war with complete information. In this sense, incomplete information still leads to war, but can no longer explain why a war does not end. Additionally, handicap signaling lends support to a rival theory of war termination that maintains that wars end when the commitment problem inherent in war is resolved (Powell 2013; Jordan 2016). According to this argument, war creates endogenous shifts in the balance of power that might be reversed by the onset of peace. Combatants who might benefit from these shifts have an incentive to agree to bargains and then seek to revise them later. Since countries may resume hostilities after the onset of these power shifts, states may choose to continue fighting in the presence of complete information. Handicaps might aggravate such commitment problems if the effects of a handicap were reversed by the onset of peace.

### Parity

An important branch of international relations literature explores whether countries are more likely to fight when they approach parity in strength. Empirical work has produced mixed evidence as to whether there is a relationship between parity in strength and conflict (Bremer 1992; Gibler 2017). This underlines the need for a careful inspection of models of interstate bargaining models to determine whether or not there is a theoretical foundation for such a relationship. For example, Slantchev (2005) has found a non-monotonic relationship between power and war in a model where states choose an endogenous and observable level of military power to signal resolve. For Slantchev, war is most likely to occur at moderate levels of strength because the strongest levels of arm deter, while the lowest levels of arm indicate a non-serious threat and insufficient resolve. Reed (2003) argues that as countries approach parity, there will be a corresponding increase in uncertainty that leads to an increase in wars..

Handicaps contribute to this discussion by expanding the micro-foundations of the theory to environments where countries can signal strength. The model presented above predicts a non-monotonic relationship between the Challenger's strength and the probability of war. Illustrated in Figure 5, this relationship is the product of two competing trade-offs. Initially, the risk of war is increasing in strength for those types who separate by making unique demands. This is a result of the risk reward trade-off, whereby increasingly stronger types make larger demands and are more willing to risk war. By contrast, the strongest types pool on the highest demand rendering the risk-reward trade-off irrelevant. Instead, these types experience a diminishing risk of war as their strength increases due to their use of handicap signals. The non-monotonic relationship between strength and war in the model supports the argument that war is most likely as countries approach parity in strength.

### Mutual Optimism

The literature on mutual optimism encapsulates much of the research on crisis bargaining with private information regarding strength. Mutual optimism is a moniker used to describe wars that begin when states have private information over strength and both believe that they can obtain higher utility from a war than from a peaceful bargain (Ramsay 2017). Studies of mutual optimism have largely focused how to properly formalize this definition and determining whether or not mutual optimism is a rational cause of war (Debs forthcoming, Fey and Ramsay 2007; 2016; Slantchev and Tarar 2011). When it comes to addressing this uncertainty, this literature has been mostly restricted to how states can use bargaining strategies to engender a risk-reward trade-off and in so doing, reveal strength by making large demands. By contrast, handicap signaling opens up a new avenue of research into mutual optimism by demonstrating that optimistic countries should be able to convey that optimism with costly signals.

## Conclusion

In this article, I argued that handicaps can serve as a signal of strength. Across modeling environments explored above and in the online appendix, the essential features of the equilibria remain consistent. First, the weakest types choose to pool on no handicaps. Second, a continuum of the strongest types adopt increasingly higher handicaps. Third, these increasingly higher handicaps are more likely to deter a rival. Finally, the common necessary condition across models is that stronger types be better able to bear the burden of handicaps than weaker types. To the best of my knowledge, this paper presents the first model of a signal of strength in international relations that allows for all types to separate.

Future work should continue to study additional potential signals of strength. Such signals are important because their theoretical predictions can have implications extend beyond the study of interstate communication. Just as handicaps have implications for theories of war termination, power transition theory and mutual optimism, so too could other signals have implications unknown.

# Appendix

## Proof of Proposition 1

The following lemma demonstrates that the Challenger's preferences are concave and is required for the proof of Proposition 1.

**Lemma 1**

*The Challenger has a concave utility function.*

*Proof*: The second derivative of equation 1 is given by

$$\psi''(x)(x - p(s_1) + c_1) + 2\psi'(x)$$

which, when substituting for $x$ as given by $\sigma_x^*$ in equation 2 becomes

$$\psi''(x)(c_1 + c_2) + 2\psi'(x)$$

To see that this is negative, recall that the Defender's strategy satisfies

$$\frac{\psi'(x)}{\psi(x)} = -\frac{1}{c_1 + c_2}$$

So that taking the derivative with respect to $x$ we find that

$$\psi''(x) = -\frac{\psi'(x)}{c_1 + c_2}$$

if we substitute this into the Challenger's second order condition, we are left simply with

$$\psi'(x)$$

which is negative as desired. ■

The remainder of the proof of Proposition 1 will proceed by demonstrating each of the claims in turn.

*Proof of claim (i)*: First, it is necessary to show that no player could benefit by deviating from the strategy profile. Since the Defender is indifferent for any demand given $\sigma_x^*$, it is only necessary to check deviations for the Challenger. Lemma 1 achieves this by demonstrating that the Challenger's utility function in 1 is concave so that the first order condition produces a maximum when the Defender plays $\psi^*(x)$. This implies that the separating types demanding $x = p(s_1) + c_2$, would never want to deviate to a demand made by another separating type. The single-crossing condition, see footnote 11, rules out the remaining possibility of separating types deviating to $x = 1$ or of types demanding $x = 1$ deviating to a demand made by a lower types and completes the proof of part (i).

*Proof of claim (ii)*: The proof proceeds in four steps. First, I identify off-path demands in any alternate equilibrium. Second, I show that D1 requires that the Defender must believe that any deviation to such an off-path belief must be conducted by the strongest type pooling on a lower demand. Third, I show that given these beliefs, it is possible to find a deviation that is preferable to the postulated equilibrium strategy. Fourth, I show that D1 does not eliminate the equilibrium described in Proposition 1.

*Step 1:*Any other Perfect Bayesian Equilibrium must have the Defender pooling on some demand $\widehat{x} < 1$. Let $\breve{s}_1$ and $\widehat{s}_1$ denote the weakest and strongest type pooling on $\widehat{x}$. The single-crossing property requires that types of the Challenger pooling on $\widehat{x}$ form a connected interval $[\breve{s}_1, \widehat{s}_1]$. Moreover, if $\widehat{x}$ is not the largest demand issued in equilibrium, let $x'$ denote the lower bound of demands larger than $\widehat{x}$. We can observe two facts about the equilibrium. First, $\widehat{x} < \sigma_x^*(\widehat{s}_1)$. If $\sigma_x^*(\widehat{s}_1) = 1$, then the claim is trivially true. Otherwise, $\sigma_x^*$ must have $\widehat{s}_1$ separate and demand $p_1(\widehat{s}_1) + c_2$. The Defender is only indifferent in response to $\widehat{x}$ if it would strictly prefer to fight type $\breve{s}_1$ and concede to $\widehat{s}_1$ so that $\widehat{x} < p_1(\widehat{s}_1) + c_2$. Second, if $x'$ exists then, $\sigma_x^*(\widehat{s}_1) \leq x'$. If $x'$ is a demand that is being pooled upon, then type $\widehat{s}_1$ must be indifferent between demanding $\widehat{s}_1$ and $x'$. If $x' = 1$, then $\widehat{s}_1 = \rho$, or the Defender would not be indifferent in response to $x'$, in which case $\sigma_x^*(\widehat{s}_1) \leq x'$ holds with equality. Otherwise, $x' < 1$ and $\sigma_x^*(\widehat{s}_1) = p_1(\widehat{s}_1) + c_2$. Once again, the Defender can only be indifferent in response to $x'$ if it would strictly prefer to fight type $\widehat{s}_1$ under complete information. This implies that $\sigma_x^*(\widehat{s}_1) < x'$. If $x'$ is not a demand being pooled upon in equilibrium, then types in the neighborhood of $\widehat{s}_1$ must be separating and playing $\sigma_x^*$ so that $\sigma_x^*(\widehat{s}_1) = x'$ is an off-path demand. Combining these two facts, we can conclude that all demands in the range $(\widehat{x}, \sigma_x^*(\widehat{s}_1))$ must be off the equilibrium path. This implies that it is appropriate to apply D1 when considering deviations to demands $x'' \in (\widehat{x}, \sigma_x^*(\widehat{s}_1))$.

*Step 2:* I will now show that the Defender must believe that any deviation to an off-path demand $x'' \in (\widehat{x}, \sigma_x^*(\widehat{s}_1))$ must be by type $\widehat{s}_1$. D1 requires that the Defender believe that the deviating type is that which is willing to deviate to $x''$ for the largest set of the Defender's possible responses. Thus it is necessary to show that type $\widehat{s}_1$ is willing to to deviate to $x''$ for a higher level of risk $1 - \phi(x'')$ than any other type. Before beginning the proof, it is useful to note that the Challenger's expected utility function satisfies the single-crossing property so that

$$U_1(\bar{x}, \phi(\bar{x})|s_1) - U_1(\underline{x}, \phi(\underline{x})|s_1) \tag{22}$$

is strictly increasing in $s_1$ for all $\bar{x} > \underline{x}$ so long as $\phi(\bar{x}) > \phi(\underline{x})$ (Ashworth and Bueno de Mesquita 2006). Following a similar logic to Dal Bó and Powell (2009, Lemma 2), this implies that whenever $U_1(x'', \phi(x'')|s_1') - U_1(\widehat{x}, \phi(\widehat{x})|s_1') = 0$ for a type $s_1' < \widehat{s}_1$ it must be the case that $\widehat{s}_1$ strictly prefers $x''$. Any weaker type $s_1^- < \check{s}_1$, must weakly prefer their equilibrium demand $x^-$ to $\widehat{x}$ implying that $U_1(x'', \phi(x'')|s_1^-) - U_1(x^-, \phi(x')|s_1^-) < 0$. This suffices to show that $\widehat{s}_1$ will prefer the deviation for a larger set of responses than any weaker type.

Next, consider the case where $x'$ exists. Recall that type $\widehat{s}_1$ must be indifferent between demanding $\widehat{x}$ and $x'$ when $x'$ is a demand that is being pooled upon. Otherwise, it must be the case that types in the neighborhood of $\widehat{s}_1$ that are stronger than it are separating. For this strategy to be incentive compatible, the Defender's strategy must feature a discontinuity between $\widehat{x}$ and $x'$ that leaves $\widehat{s}_1$ indifferent between demanding $\widehat{x}$ and $\sigma_x^*(\widehat{s}_1)$ (even if the latter demand is off-the equilibrium path) and then decrease its rate of acceptance for higher demands at a rate given by equation 6. Once again, equation 22 being increasing in type implies that whenever $U_1(x', \phi(x')|s_1'') - U_1(x'', \phi(x')|s_1'') = 0$ for a type $s_1'' > \widehat{s}_1$, type $\widehat{s}_1$ must strictly prefer $x''$. Moreover, any type stronger than $\widehat{s}_1$ issuing a demand $x^+$ larger than $x'$ must receive a weakly higher equilibrium utility for that demand then for $\widehat{x}$ so that $U_1(x^+, \phi(x')|s_1'') - U_1(x'', \phi(x'')|s_1'') > 0$. This suffices to show that $\widehat{s}_1$ will be prefer the deviation for a larger set of responses than any stronger type.

*Step 3:* Following Dal Bó and Powell (2009, Lemma 3), it is straightforward to show that the Challenger has a profitable deviation to an off-path belief. If $\widehat{s}_1 \neq \rho$, then any deviation to $x'' \in (\widehat{x}, \sigma_x^*(\widehat{s}_1))$, must have the Defender strictly prefer to concede to the Challenger in response to $x''$ than fight. If $\widehat{s}_1 = \rho$, then the same is true only for demands $x'' \in (\widehat{x}, p_1(\rho) + c_2)$. Regardless, in both cases such such a deviation simultaneously increases the size of the demand while reducing the risk of war, implying that the Challenger must strictly prefer it in response to all of of the Defender's best responses. This is sufficient to show that the alternate equilibrium is unstable under D1.

*Step 4:* Finally, D1 does not eliminate the equilibrium described in Proposition 1. This is because the arguments in Step 2 imply that the Defender will believe that a deviation to an off-path demand $x'' \in (p_1(\rho) + c_2, 1)$ must be performed by type $\rho$. The Defender would strictly prefer to fight type $\rho$ in response to any such demand and will only play $\phi(x'') = 0$.

Therefore, no type can increase their expected utility by deviating to any off-path demand $x'' \in (p_1(\rho) + c_2, 1)$. ■

## Proof of Proposition 2

As in Proposition 1, the proof of Proposition 2 requires that the Challenger have a concave utility function which also satisfies single-crossing. The conditions required for these properties are recovered in the following lemmata.

**Lemma 2**

*The signaling country's payoffs are concave provided that equation 10 is satisfied.*

*Proof*: To demonstrate that equation 10 is sufficient for concavity, take the second derivative of the Challenger's payoffs as given in equation 7.

$$\phi''(h)(1 - p_1(s_1, h) + c_1) + \frac{\partial^2 p_1(s_1, h)}{\partial^2 h}(1 - \phi(h)) - 2\phi'(h)\frac{\partial p_1(s_1, h)}{\partial h} < 0$$

Note that nothing can be said regarding whether or not this equation holds without first examining $\phi''(h)$ in greater detail. To find $\phi''(h)$ I rewrite the hazard rate function as follows

$$\phi'(h)(1 - p_1(\sigma_h^{-1}(h), h) + c_1) = -\frac{\partial p_1(\sigma^{-1}(h), h)}{\partial h}(1 - \phi(h))$$

From the Defender's perspective, in a separating equilibrium, as $h$ increases so should $s_1$ because $s_1 = \sigma^{-1}(h)$. This makes this derivative of $\phi'(h)$ different from the second derivative of the Challenger's expected utility function because the Challenger has a fixed type regardless of its choice of handicap. Taking this derivative one obtains that

$$\phi''(h)(1 - p_1(s_1, h) + c_1) - \frac{\phi'(h)}{d\sigma_h(s_1)}\frac{\partial p_1(s_1, h)}{\partial s_1} - \phi'(h)\frac{\partial p_1(s_1, h)}{\partial h} =$$
$$-\frac{1 - \phi(h)}{d\sigma_h(s_1)}\frac{\partial^2 p_1(s_1, h)}{\partial s_1 \partial h} - \frac{\partial^2 p_1(s_1, h)}{\partial^2 h}(1 - \phi(h)) + \phi'(h)\frac{p_1(s_1, h)}{\partial h}$$

Next observe that since it must be the case that $c_2 = p_1(\sigma_h^{-1}(h), h)$ taking the derivative of $p_1(s_1, h)$ with respect to $h$ one obtains the following useful identity

$$0 = \frac{\partial p_1(s_1,h)}{\partial s_1}\frac{1}{d\sigma_h(s_1)} + \frac{\partial p_1(s_1,h)}{\partial h} \tag{23}$$

Substituting this identity in on the left-hand side $\phi''(h)$ becomes

$$\phi''(h)(1-p_1(s_1,h)+c_1) = -\frac{1-\phi(h)}{d\sigma_h(s_1)}\frac{\partial^2 p_1(s_1,h)}{\partial s_1 \partial h} - \frac{\partial^2 p_1(s_1,h)}{\partial^2 h}(1-\phi(h)) + \phi'(h)\frac{p_1(s_1,h)}{\partial h} \tag{24}$$

Substituting in this value for $\phi''(h)$ back into the Challenger's second derivative and rearranging the inequality becomes

$$\frac{1}{d\sigma_h(s_1)}\frac{\partial^2 p_1(s_1,h)}{\partial s_1 \partial h} > \left(\frac{\partial p_1(s_1,h)}{\partial h}\right)^2 \frac{1}{c_1+c_2}$$

Then adding and subtracting $\frac{1}{c_1+c_2}\frac{\partial p_1(s_1,h)}{\partial s_1}\frac{\partial p_1(s_1,h)}{\partial h}\frac{1}{d\sigma(s_1)}$ to the right-hand side the inequality becomes

$$\frac{1}{d\sigma(s_1)}\frac{\partial^2 p_1(s_1,h)}{\partial s_1 \partial h} > \frac{\frac{\partial p_1(s_1,h)}{\partial h}}{c_1+c_2}\left[\frac{\partial p_1(s_1,h)}{\partial h} + \frac{\partial p_1(s_1,h)}{\partial s_1}\frac{1}{d\sigma(s_1)}\right] - \frac{1}{d\sigma(s_1)}\frac{\frac{\partial p_1(s_1,h)}{\partial s_1}\frac{\partial p_1(s_1,h)}{\partial h}}{c_1+c_2}$$

Then applying equation 23 and dividing both sides by $\frac{1}{d\sigma_h(s_1)}$ we find the inequality in equation 10. ■

**Lemma 3**

*The Challenger's utility function satisfies the single-crossing condition provided that 10 holds.*

*Proof:* The goal of this proof is to find conditions under which

$$\phi(h) + (1-\phi(h))(p_1(s_1,h) - c_1) \geq \phi(h') + (1-\phi(h'))(p_1(s_1,h') - c_1)$$

holds for $s_1$ when $h$ is given by $\sigma_h^*(s_1) = h > h'$ and

$$\phi(h') + (1 - \phi(h'))(p_1(s_1', h') - c_1) \geq \phi(h) + (1 - \phi(h))(p_1(s_1', h) - c_1)$$

holds for $s_1'$ when $\sigma_h(s_1') = h' < h$. That is a type $s_1$ prefers a handicap of $h$ given by $\sigma_h^*$ to any lower handicap and similarly if a type $s_1'$ prefers a handicap $h'$ given by $\sigma^*$ to any higher handicap. Adding the inequalities one finds that

$$(1 - \phi(h))(p_1(s_1, h) - p_1(s_1', h)) \geq (1 - \phi(h'))(p_1(s_1, h') - p_1(s_1', h')$$

Rearranging, the inequality becomes

$$\begin{aligned}(1 - \phi(h))\{[p_1(s_1, h) - p_1(s_1', h)] - [(p_1(s_1, h') - p_1(s_1', h')]\} \\ \geq (\phi(h) - \phi(h'))[(p_1(s_1, h') - p_1(s_1', h')]\end{aligned}$$

Then dividing both sides by the change in both variables

$$\begin{aligned}\frac{(1 - \phi(h))\{[p_1(s_1, h) - p_1(s_1', h)] - [(p_1(s_1, h') - p_1(s_1', h')]\}}{\Delta s_1 \Delta h} \\ \geq \frac{(\phi(h) - \phi(h'))}{\Delta h} \frac{[(p_1(s_1, h') - p_1(s_1', h')]}{\Delta s_1}\end{aligned}$$

Which can be rewritten as (Marsden and Tromba 1996, 173-174)

$$(1 - \phi(h))\frac{\partial^2 p_1(s_1, h)}{\partial s_1 \partial h} \geq \phi'(h)\frac{p_1(s_1, h)}{\partial s_1}$$

Substituting in for the hazard rate as defined in equation 14, one obtains

$$\frac{\partial^2 p_1(s_1, h)}{\partial s_1 \partial h} \geq -\frac{\partial p_1(s_1, h)}{\partial s_1}\frac{\partial p_1(s_1, h)}{\partial h}\frac{1}{c_1 + c_2}$$

as desired. ■

The remainder of the proof of Proposition 2, will proceed by demonstrating each of the claims in turn.

*Proof of claim (i):* The Challenger's expected utility function being concave (Lemma 2) ensures that there exists a unique maximum given by $\sigma_h^*$ when the Defender plays $\phi^*$ as it is derived from first order conditions. Therefore no type adopting an interior solution, that is a signal $h > 0$, would choose to deviate to any other signal $h' > 0$. Lemma 3 then demonstrates that the optimal handicap for the Challenger is weakly increasing in type, ruling out a deviation to the corner solution of $h = 0$. It also rules out deviations by those adopting $h = 0$ to a handicap signal $h > 0$. This rules out any potential deviations by the Challenger to any other on-the-path signal.

It remains to consider possible deviations by the Defender. Note that if the Challenger plays according to $\sigma_h^*$ then for all handicap signals greater than 0, the Defender is indifferent between standing firm and backing down and so there can be no profitable deviation. Moreover, the Defender's strategy for fighting in response to a handicap signal $h = 0$, is strictly greater than its payoff for backing down in which case it receives a payoff of zero. Thus the Defender has no incentive to deviate from $\phi^*$.

*Proof of claim (ii):* I now demonstrate that $\sigma_h^*$ is the unique signaling function that allows for types $s_1 > \widetilde{s}_1$ to separate. Let $\overline{h} \equiv \sigma^*(\bar{s_1})$ denote the handicap signal used by the strongest signaling type according per $\sigma^*$ in equation 11. Note, that when the strongest type adopts this handicap, their probability of winning a war is $p_2(\bar{s_1}, \overline{h}) = c_2$. No type can adopt a handicap larger than $\widehat{h}$ as this would leave the the Defender with a payoff to fighting that is larger than 0 regardless of the signaling type. This implies that the Defender has a strictly dominant strategy to stand firm for any belief. Then, because handicaps are costly, if the Defender is sure to stand firm, no type of the Challenger will find it optimal to deviate to a handicap $h > \bar{h}$. This implies that no alternative signaling strategy may use handicap signals off-the-equilibrium path of $\sigma_h^*$. Similarly, any alternative signaling strategy that requires a type $s_1 > \widetilde{s}_1$ to separate using a signal other than that stipulated by $\sigma_h^*$ cannot leave the

Defender indifferent. Therefore, $\sigma_h^*$ must be the only signaling strategy that where all types $s_1 > \widetilde{s}_1$ separate.

*Proof of claim (iii)*: The proof of this claim follows identical steps to the ruling out of alternate equilibria as it did in Proposition 1. ■

## Proposition 3

The proof relies on the following lemma which demonstrates that the Challenger's expected utility is maximized by a demand that is a "corner" solution. For convenience, the subscript $i$ in $\varphi_i(x, h(x))$ will denote the derivative of $\varphi(x, h(x))$ with respect to its $i - th$ argument.

**Lemma 4**

*The Challenger's expected utility function in equation 21 has no interior maximum and is instead maximized by one of the two "corner" solutions* $x = p_1(\underline{s}_1) + c_2$ *or* $\min\{p_1(s_1, 0) + c_2, 1\}$

*Proof*: To prove this claim, I demonstrate the first order condition generates a saddle point. Taking the first order condition of equation 21 with respect to $x$, we find that

$$1 + \left[\varphi_1(x, h^*(x)) + \varphi_2(x, h^*(x))\frac{dh^*(x)}{dx}\right](c_1 + c_2) = 0$$

Note that since it must be the case that $p_1(s_1, h^*(x)) = x - c_2$, then $\frac{dh^*(x)}{dx} = \frac{1}{\frac{\partial p_1(s_1, h^*(x))}{dh^*(x)}}$. So substituting in for $\frac{dh}{dx}$ and $\varphi_2(x, h^*(x))$ from equation 14 one finds that

$$\frac{\varphi_1(x, h^*(x))}{\varphi(x, h^*(x))} = -\frac{1}{c_1 + c_2}$$

Examining the second order condition of equation 21 with respect to $x$, we find that

$$\left[\varphi_{11}(x, h^*(x)) + \varphi_{12}(x, h^*(x))\frac{dh^*(x)}{dx}\right](c_1 + c_2) + \varphi_1(x, h^*(x)) + \varphi_2(x, h^*(x))\frac{dh^*(x)}{dx}$$

Substituting in for $\frac{dh^*(x)}{dx}$, $\varphi_1(x, h^*(x))$ and $\varphi_2(x, h^*(x))$, this expression simplifies to

$$\left[\varphi_{11}(x, h^*(x)) + \varphi_{12}(x, h^*(x))\frac{1}{\frac{\partial p_1(s_1, h^*(x))}{dh^*(x)}}\right](c_1 + c_2) - \frac{1}{c_1 + c_2}$$

To find expressions for $\varphi_{11}$ and $\varphi_{12}$ it is necessary to take the derivative with respect to $x$ of $\varphi_1(x, h^*(x))$. Following similar steps, one finds that this results in

$$\left[\varphi_{11}(x, h^*(x)) + \varphi_{12}(x, h^*(x))\frac{1}{\frac{\partial p_1(s_1, h^*(x))}{dh^*(x)}}\right](c_1 + c_2) = \frac{1}{c_1 + c_2}$$

So the second order condition must be equal to zero. This suggests that the first order condition produces a saddle point when both the choice of handicap and demand are both interior solutions.

This is verified by an inspection of the effects of an increase in $x$ on equation 21. An increase in $x$ leads to a linear increase in the first term, but a convex decrease in the second. It follows that one effect must dominate the other. If the linear increase in $x$ overtakes the effect of the decrease in $\varphi(x, h^*(x))$, then the Challenger will make the largest demand they can make under which they can leave the Defender indifferent between accepting and not. For types $s_1 > \widetilde{s}_1$ this will involve the use of handicap signals. If the convex increase in $\varphi(x, h^*(x))$ from lower demands overtakes the linear effect of decreasing $x$, then the Challenger reduces their demand as much as possible. The lowest demand the Challenger can make is the that made by type $\underline{s}_1$ at which point the effects of handicapping no longer matter as this demand can be accepted with certainty. ■

*Proof of claim (i)*: When the Challenger plays $\sigma^*$ as in equations 16 and 11, then the Defender is always indifferent in response to the Challenger's choice of demand and handicap. Therefore, it is only necessary to check that the Challenger is playing a best response. Per the previous lemma, the Challenger will either demand $x = p_1(\underline{s}_1) + c_2$ or $x = \min\{p_1(s_1, 0) + c_2, 1\}$. However, Proposition 1 demonstrated that the latter is always preferable even if $x = p_1(\underline{s}_1) + c_2$ is accepted with probability 1. It follows that the Challenger issues demands according to equation 19. Lemma 4 rules out both $x$ and $h$ being interior solutions and these types do not signal. The Challenger's signaling strategy being a best response for types pooling on the demand $x = 1$ follows the same arguments as in Proposition 2, part (i). Again Lemma 4 rules out the possibility that signaling types pooling on $x = 1$ have a profitable deviation shifting to a lower demand. This suffices to prove the claim.

*Proof of claim (ii)*: The proof of this claim follows analogous steps to those in Proposition 1.

■

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