**Online Supplemental Appendix**

Signaling Strength with Handicaps

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# Contents

# A Handicap Signaling in the US Civil War

This section contains supporting information about the case presented in the introduction of the main text. It includes (1) a discussion of the evolution of British policy towards the US Civil war and of how the British came close to the brink of intervening in the conflict in the summer of 1862. This contributes to the argument that handicap signaling played an important role in deterring intervention. This section also includes (2) a short discussion of the evidence in favor of handicap signaling being the cause for the delay of the Emancipation Proclamation while also addressing electoral considerations as a plausible alternative explanation.

## A.1 British Policy Towards The US Civil War

In the initial stages of the US Civil War, France and Great Britain sought to jointly intervene in the conflict on behalf of the Confederacy. Though staunch abolitionists, their support for the Union was undermined when Lincoln insisted that the war was being fought to preserve the Union and not to end slavery. Perplexed, British leadership took Lincoln at his word, failing to understand that his posturing was designed to prevent additional secession. Instead, Britain and France accepted Confederate claims that the Confederacy was fighting a just war for self-determination, a narrative that suited their interests in weakening an aggressive United States and their need for cotton.[1]

[1] See Jones 1992 15, 34; Jones 2010, 28-31, 55, 69, 120-124.

Britain decided to wait before intervening until the Union had been sufficiently weakened by the Confederacy.[2] The Union repeatedly threatened that any British involvement in the conflict would lead to war.[3] British leadership took this threat seriously. The Union had amassed a large army, boasted a powerful navy, and its proximity to Canada posed a serious risk (Jones 2010, 95). Yet British Prime Minister Palmerston and Foreign Secretary Russell also believed that the Confederacy would ultimately prevail - the Union could not conquer so vast a territory.[4] British policy sought to minimize the risk of war with the Union by letting the Confederacy exhaust it first. In a speech to parliament, Prime Minister Palmerston resisted calls for intervention and insisted that Britain would only become involved once Confederate independence had been "firmly and permanently" established through battlefield success (Jones 2010, 172). However, in the summer of 1862, as cotton shortages caused pressures for intervention to mount, the Army of the Potomac suffered a defeat at the Seven Days' Battle (June 25th-July 1st). As discussed in the main text, this defeat robbed the Union of the initiative meant that the Union could not take the Confederate capital at Richmond and a bring about a quick end to the war.

Additional Union defeats brought Great Britain close to the brink of intervention. Shortly after the Lincoln decided to delay the Proclamation, the Confederacy won a major victory at the Second Battle of Bull Run (August 30th). This defeat forced the Army of the Potomac to retreat towards Washington and left a path open for a Confederate invasion of Maryland. Upon hearing of these events, Prime Minister Palmerston reveled in the Union's dire straits and wrote that an additional Union defeat would suffice to trigger European involvement (Jones 2010, 215-219). However, the Union victory at Antietam reversed Palmerston's position: he would refrain from intervention so long as the Union threatened to fight in response (Jones 2010, 233; Foreman 2010, 317-318). Given the strong British reaction to the Emancipation Proclamation and its perception as a desperate wartime measure, both by the British

[2] France, though eager, repeatedly made intervention contingent on British involvement. This implied that it was British opinion that mattered. See Jones 2010, 131-138, 163-169, 176-179; Jones 1992, 74.

[3] Jones 2010, 37-38, 41-42, 50, 58-59, 151, 201.

[4] Jones 2010, 1, 32-33, 38, 60-61; Jones 1992, 22-23, 54, 57-58.

public and by members of the British cabinet, it appears likely that Seward's decision to postpone the Proclamation helped thwart an intervention (Jones 1992, 179).

## A.2 Alternative Explanations

Some time after the conclusion of the July 22nd Cabinet meeting Lincoln decided to delay issuing the Emancipation Proclamation. Though his cabinet raised several objections to the Proclamation, only Seward's arguments received Lincoln's express approval.[5] Moreover, when Lincoln finally did decide to issue the Proclamation on September 22nd he justified the decision to the cabinet by reminding them that he had decided to hold off on issuing the Proclamation until after the Union had won a victory, just as Seward had suggested (Guelzo 2004, 169-171).

However, this does not rule out the possibility that other causes could have contributed to delaying the Proclamation. Another plausible explanation is that Lincoln chose to delay the Proclamation due to electoral concerns. The Union's numerous military failures and the unpopular draft loomed over the coming election. The Emancipation Proclamation threatened to further undermine Republican support (McPherson 1988, 560-562). Opposition to Emancipation was central to the Democratic platform and congressional votes on antislavery measures during the early stages of the war were split along party lines (McPherson 1988, 506-508). When Lincoln reconvened the Cabinet on September 22nd to notify it of his intention to issue the Proclamation, Postmaster-General Blair reminded Lincoln that the Proclamation could energize secessionists in the border states and strengthen support for the Democratic party (McPherson 1988, 557).

Ultimately, Lincoln chose to dismiss these electoral concerns. Lincoln elected to proceed with the Proclamation over Blair's objections, telling Blair that the border states had had their chance and that the Democrats would criticize his administration regardless. Moreover, in addition to Lincoln's rejection of electoral considerations, this explanation cannot make

[5] Lincoln is later recorded as confiding that "the wisdom of the view of the Secretary of State struck me with very great force" (Guelzo 2004, p. 137).

sense of the length of the delay. If Lincoln had decided to delay for electoral reasons it seems unlikely that he would delayed the Proclamation only to release it prior to the 1862 elections anyway.

# B Handicap Signaling to Deter Third-Party Intervention

Whereas the model in the main text investigates handicap signaling in a two-player game, the US Civil War case is an example of handicapping to deter third-party intervention. In this subsection, I analyze a model of handicap signaling that more closely resembles this strategic environment. The model is a modified version of Kydd and Straus (2013) and begins with a bargaining stage in which a Challenger with private information regarding its strength issues an ultimatum to a Defender. Should the states fail to come to an agreement, a war begins and the game proceeds to a second stage in which the Challenger can handicap itself to try and deter a third party from intervening on the Defender's behalf.

The results are qualitatively similar to those in the main text. Moderately resolved types can bluff in the bargaining stage, taking advantage of the Defender's hesitancy to fight the strongest types by mimicking their demands. As a result, the third party will be unsure of the Challenger's type once a war begins. If the third party would rather concede to some of these types of the Challenger but fight others, then strong types of the Challenger can benefit from revealing their strength by issuing handicap signals. As in the main text, signaling types will separate with stronger types adopting larger handicaps. The third party can then infer that a larger handicap implies a stronger signaler and is in turn more likely to be deterred.

## B.1 Model Setup

Suppose that two countries are engaged in an ultimatum game in which a third party may choose to intervene. In the first stage of the game a Challenger makes a demand $x \in [0,1]$ of the Defender. The Defender can either accept the offer and receive $1-x$ ending the game, or reject the offer and go to war in which case the game continues. In the second stage of the game, the Challenger attempts to deter the third party by handicapping itself. Specifically, the Challenger selects a handicap $h$, which strictly reduces its chance of winning a war against the Defender. The third party observes both the Challenger's choice of demand and its choice of handicap and then decides whether to intervene on behalf of the Defender. For simplicity's sake, I assume that the third party has identical interests to those of the Defender regarding the distribution of the good.[6]

War takes the form of a costly lottery function which determines ownership over the good being bargained over. In a bilateral war between the Challenger and the Defender, the Challenger has probability of winning $p_1(s_1, h)$ which is strictly increasing in its strength $s_1$ and strictly decreasing in its choice of handicap $h$. Stronger types are better able to bear the burden of handicaps, and $p_1(s_1, h)$ satisfies the increasing differences assumption laid out in Assumption 1 in the main text. If the third party chooses to intervene on behalf of the Defender, then the Challenger must separately defeat both the Defender and the third party to win the war. The third party's probability of winning a conflict with the Challenger is $p_3(s_1)$ which is strictly decreasing in $s_1$. Putting these components together, the Challenger's combined probability of winning a trilateral conflict is $p_1^c(s_1, h) = p_1(s_1, h)(1 - p_3(s_1))$. The Challenger has private information regarding its strength $s_1$ which is distributed according to the common knowledge, continuous, and strictly increasing CDF $F$ with support $[\underline{s}_1, \bar{s}_1]$.

All countries can expect to pay a cost for fighting. The Challenger incurs higher costs of fighting in a trilateral war $c_1^t$ than in a bilateral war with the Defender alone $c_1^b$ ($c_1^t \geq c_1^b$).

[6] For example, if the game ends because the Defender accepts the offer, then the third party also receives a utility of $1 - x$.

Conversely, the Defender pays a smaller sum for fighting in a trilateral war $c_2^t$ than for fighting in a bilateral war $c_2^b$ ($c_2^b \geq c_2^t$). Meanwhile the third party only pays its cost for fighting $c_3$ if it chooses to intervene.

Given this setup, a strategy for the Challenger is a mapping from its strength to a choice of demand in stage 1 and a choice of handicap in stage 2, $\sigma : s_1 \rightrightarrows [0,1] \times [0, \bar{h}]$. The Defender will only get to observe the Challenger's demand before acting. Therefore, it will have posterior beliefs $G(s_1|x)$ and its strategy will be a mapping from the Challenger's demand to the probability that it will accept the offer $\psi : x \to [0,1]$. On the other hand, the third party will observe both the Challenger's choice of demand and handicap before acting. Therefore, the third party will have posterior beliefs $J(s_1|x,h)$ and its strategy will be a mapping from the Challenger's demand and handicap to the probability that it will stay out of the war, $\phi : x \times h \to [0,1]$. Putting it all together, the Challenger's expected utility function is given by

$$x\psi(x) + (1-\psi(x))[\phi(h)(p_1(s_1,h) - c_1^b) + (1-\phi(h))(p_1(s_1,h)(1-p_3(s_1)) - c_1^t) \quad \text{(B. 1)}$$

As in the main text, I will characterize Perfect Bayesian Equilibria.

## B.2 An Assumption About the Benefits of Intervention

It is important to note that under this setup the benefit to intervention for the third party can be *increasing* in the Challenger's strength. Formally, the benefit to the third party for intervention is given by

$$p_1^b(s_1, 0)p_3(s_1) - c_3 \quad \text{(B. 2)}$$

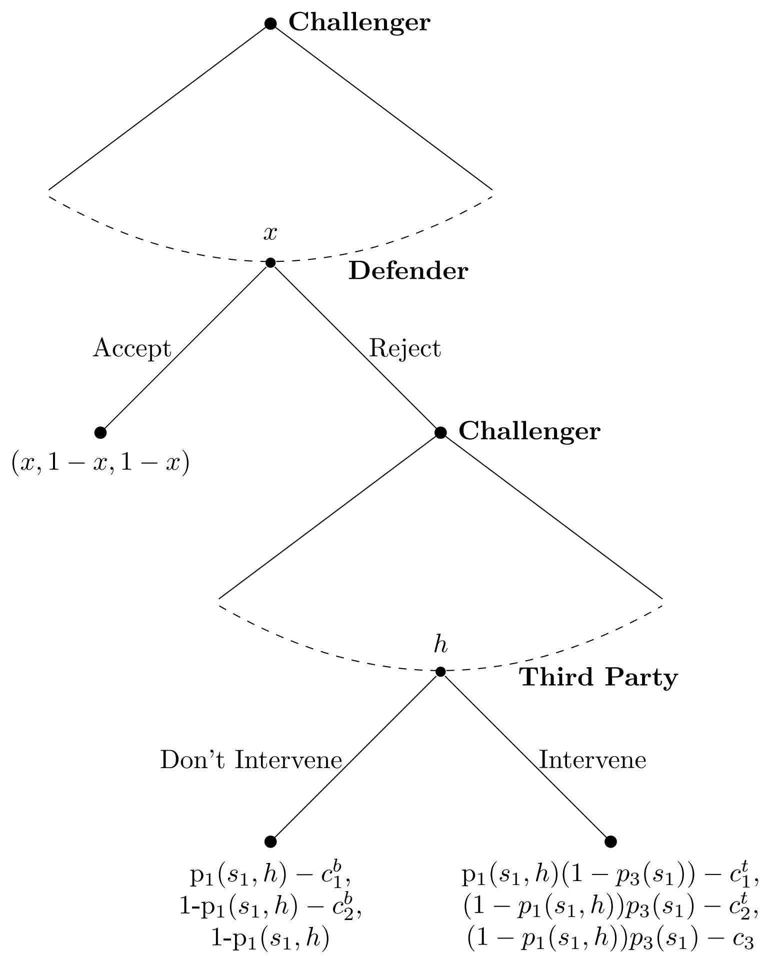


Figure B.1: **Handicap Signaling to Deter Third-Party Intervention:** This figure depicts the game tree in an environment in which handicap signaling is designed to deter intervention in an ongoing war. The model is a modified version of Kydd and Straus (2013) and consists of two stages. First, a bargaining stage in which the Challenger makes a TIOLI demand of the Defender where rejection leads to war. Second, a handicap signaling stage in which the Challenger handicaps itself in its war with the Defender to deter a third party from intervening.

which is the probability that the third party wins its conflict with the Challenger when the Defender would have lost minus its cost of fighting. If it is the case that,

$$\frac{\frac{\partial p_1(s_1,0))}{\partial s_1}}{p_1(s_1,0)} > -\frac{\frac{dp_3(s_1)}{ds_1}}{p_3(s_1)}$$

then as the Challenger becomes stronger, its probability of defeating the Defender increases faster than its probability of defeating the third party.[7] This implies that the third party's assistance becomes more valuable against stronger types of the Challenger. I therefore impose the following assumption

**Assumption B. 1**

$$-\frac{\frac{dp_3(s_1)}{ds_1}}{p_3(s_1)} > \frac{\frac{\partial p_1(s_1,0))}{\partial s_1}}{p_1(s_1,0)} \tag{B. 3}$$

So long as this condition holds, then the third party has less to gain from intervening as the Challenger grows stronger. This is a necessary condition for handicap signaling.

## B.3 Equilibrium Characterization

The game is solved using backwards induction. I therefore begin by examining the Challenger's decision whether to handicap itself once a war has begun. From the discussion in the "Modeling Handicaps" section of the main text, we know that the third party must be playing a mixed strategy in response to any positive level of handicaps. This requires that the third party be indifferent between intervening and not. Expression B. 2 must therefore equal zero so that with separation each type of the Challenger handicaps itself down to $p_1^b(s_1, h) = \frac{c_3}{p_3(s_1)}$. It follows that only sufficiently strong types $s_1$ that can satisfy $p_1(s_1, 0) > \frac{c_3}{p_3(s_1)}$ will be able to handicap themselves. As in the main text, there exists an additional restriction on the cross partial of $p_1(s_1, h)$ that must be satisfied for handicap

[7] For example, it could be that the Challenger's strength and its war with the Defender is primarily land-based but that a war against the third party would be a naval conflict.

signaling to be viable. The following lemma establishes conditions under which the Challenger's expected utility function will be concave in $h$ and handicap signaling will be viable. (All proofs at the end of the section)

**Lemma B. 1**

*The Challenger's expected utility function will be both concave in $h$ and will satisfy single crossing if the following inequality holds*

$$\frac{\partial^2 p_1(s_1,h)}{\partial s_1 \partial h} \geq \frac{1-\phi(h)}{\phi(h)+(1-\phi(h))(1-p_3(s_1)))} \frac{dp_3(s_1)}{ds_1} \frac{\partial p_1(s_1,h)}{\partial h} + \frac{\partial p_1(s_1,h)}{\partial h} \frac{1}{c^3+c_1^t-c_1^b} \left[ \frac{dp_3(s_1)}{ds_1} p_1(s_1,h) + p_3(s_1) \frac{\partial p_1(s_1,h)}{\partial s_1} \right] \tag{B. 4}$$

Note that the terms in the square brackets on the right-hand side of the equation are positive given assumption B. 1.

Whenever the inequality in equation B. 4 holds, stronger types of the Challenger can benefit from signaling. Specifically, sufficiently strong types of the Challenger will handicap themselves up until the point where the marginal cost of doing so is equal to the marginal gain of doing so as in equation B. 5.

$$\frac{\phi'(h)}{\phi(h)+(1-\phi(h))(1-p_3(s_1)))} = -\frac{\frac{\partial p_1(s_1,h)}{\partial h}}{c_3+c_1^t-c_1^b} \tag{B. 5}$$

Here the left-hand side presents a modified "hazard rate" and is the probability that an increase in handicap $h$ will deter the third party divided by the probability that the Challenger will either deter the third party or defeat it in conflict after failing to deter it. The right-hand side represents the costs of handicapping and has a similar interpretation to its counterpart in the main text. The third party will play the strategy in B. 5 with the additional requirement that $\phi(0) = 0$. Unfortunately, this ordinary differential equation does not have a closed form solution.

Having determined the Challenger's and third party's strategies in the second stage, we can proceed to the first stage and analyze the Challenger's choice of demand and the

Defender's response. As in the main text, separation can arise because the Defender can make use of the risk-reward trade-off to deter weaker types from making larger demands. Handicap signaling is only required in the second stage if the Challenger fails to have all types separate during the bargaining stage. Otherwise the third party learns the Challenger's type in the first stage and the Challenger does not need to issue a handicap signal. Therefore handicap signaling requires that there exist types that the Defender would strictly prefer not to fight regardless of the size of their demand.

However, the existence of uncertainty in the second stage of the game is insufficient to generate handicap signaling if the third party knows that it is unwilling to fight any of the pooling types. Let $\widetilde{s}_1^b$ denote the type of Challenger who the Defender is indifferent between fighting in a bilateral war when the Challenger demands the entire good, $p_1(\widetilde{s}_1^b, 0) = 1 - c_1^b$. Moreover, let $\widetilde{s}_1^t$ denote the type of Challenger that the Defender is indifferent between fighting and conceding to in a trilateral war when the Challenger demands the entire good, $(1 - p_1(\widetilde{s}_1^t, 0))p_3(\widetilde{s}_1^t) = c_1^t$. Similarly let $\widetilde{s}_1^3$ denote the type that makes the third party indifferent between intervening on behalf of the Defender and not so that B. 2 equals 0. While it will always be the case that $\widetilde{s}_1^b > \widetilde{s}_1^t$ by assumption, there is no guarantee that $\widetilde{s}_1^3 > \widetilde{s}_1^t$. Moreover, even when it is the case that $\widetilde{s}_1^3 > \widetilde{s}_1^t$, the Defender may still be unwilling to risk fighting the Challenger when she fails to separate. As in the main text, when the Challenger fails to separate, there will be a set $[\rho, \widetilde{s}_1^t)$ or $[\rho, \widetilde{s}_1^b)$ of bluffing types. If the types of the Challenger who pool on $x = 1$ can deter the third-party without a handicap, then they will not signal.

**Lemma B. 2**

*There cannot be a PBE with handicap signaling if the following inequality does not hold*

$$\int_{\rho}^{\bar{s}_1} p_1(s_1, 0)p_3(s_1) - c_3 > 0 \tag{B. 6}$$

The inequality is an analogue to equation 9 from the main text hold.

On the other hand, whenever the mass of pooling types do not deter the third-party, then the Challenger will handicap itself to do so.

**Proposition B. 1**

*If equations B. 4, B. 3 and $\widetilde{s}_1^t < \widetilde{s}_1^3$ hold, then there exists a PBE in which,*

*(i) the Challenger demands*

$$x^*(s_1) = \begin{cases} p(s_1)(1-p_3(s_1)) + c_2 & \text{if } p_1(s_1) < \rho \\ 1 & \text{if } p_1(s_1) \geq \rho \end{cases} \tag{B. 7}$$

*where $\rho$ is given by*

$$\rho \equiv \Bigg\{ s_1' : c_2 = \frac{\int_{s_1'}^{\widetilde{s}_1^3} f(s_1)p_1(s_1,0)(1-p_3(s_1))ds_1}{F(\widetilde{s}_1^3) - F(\rho)} + \frac{\int_{\widetilde{s}_1^3}^{\bar{s}_1} f(s_1)\phi(h^*(s_1))p_1(s_1,h^*(s_1)) + (1-\phi(h^*(s_1))p_1(s_1,h^*(s_1))(1-p_3(s_1))ds_1}{F(\bar{s}_1) - F(\widetilde{s}_1^3)} \Bigg\} \tag{B. 8}$$

*(ii) the Defender responds by playing*

$$\psi(x) = \begin{cases} e^{-\frac{x - p(\underline{s}_1)(1-p_3(\underline{s}_1)) - c_2^t}{c_1^t + c_2^t}} & \text{if } x < 1 \\ \frac{(c_1^t + c_2^t) e^{-\frac{p_1(\rho)(1-p_3(\rho)) - p_1(\underline{s}_1)(1-p_3(\underline{s}_1))}{c_1^t + c_2^t}}}{1 - p_1(\rho)(1-p_3(\rho)) + c_1^t} & \text{if } x = 1 \end{cases} \tag{B. 9}$$

*and has beliefs given by (5).*

*(iii) the Challenger handicaps itself according to*

$$h^*(s_1) = \begin{cases} 0 & \text{if } p_1(s_1, 0) \leq \frac{c_3}{p_3(s_1)} \\ p_1(s_1, h) = \frac{c_3}{p_3(s_1)} & \text{if } p_1(s_1, 0) > \frac{c_3}{p_3(s_1)} \end{cases} \tag{B. 10}$$

*(iv) and the third party responds by playing according to B. 5 with $\phi(0) = 0$ and has beliefs given by*

$$K^*(s_1|x,h) \equiv \begin{cases} 1 & \textit{if } s_1 = \sigma_x^{*-1}(x) \textit{ and } x < 1, h = 0 \\ \frac{f(s_1)}{F(\tilde{s})1)-F(\rho)} & \textit{if } x = 1, h = 0 \\ 1 & \textit{if } s_1 = \sigma_h^{*-1}(h) \textit{ and } x = 1, h > 0 \end{cases} \tag{B. 11}$$

## B.4 Section Proofs

### B.4.1 Proof of Lemma B. 1

The proof contains two parts. The first demonstrates that the Challenger's expected utility function is concave in $h$ and the second shows that this implies that the Challenger's expected utility function also satisfies single crossing.

*Step 1:* To check the second order condition for concavity, take the derivative of equation B. 1 twice with respect to $h$ to find that it will be concave if,

$$\phi''(h)[c_1^t + c_3 - c_1^b] + \phi'(h)\frac{\partial p_1(s_1,h)}{\partial h}p_3(s_1))$$
$$+\frac{\partial^2 p_1(s_1,h)}{\partial^2 h}[\phi(h) + (1-\phi(h)(1-p_3(s_1)] + \frac{\partial p_1(s_1,h)}{\partial h}[\phi'(h) - \phi'(h)(1-p_3(s_1))] < 0$$

Unfortunately we cannot determine whether this inequality holds without first solving for $\phi''$. Turning to the hazard rate in equation B. 5 we take its derivative with respect to $h$, while noting that when types of the Challenger separate $s_1 = \sigma^{-1}(h)$. This leaves us with

$$\phi''(h)[c_3 + c_1^t - c_1^b] + \phi'(h)\left[\left(\frac{\partial p_1(s_1,h)}{\partial s_1}\frac{1}{d\sigma(s_1)} + \frac{\partial p_1(s_1,h)}{\partial h}\right)p_3(s_1)) - p_1(s_1,h)\frac{dp_3(s_1)}{ds_1}\frac{1}{d\sigma(s_1)}\right] =$$
$$-\frac{\partial^2 p_1(s_1,h)}{\partial s_1 \partial h}\frac{1}{d\sigma(s_1)}[\phi(h) + (1-\phi(h)(1-p_3(s_1)] - \frac{\partial^2 p_1(s_1,h)}{\partial^2 h}[\phi(h) + (1-\phi(h)(1-p_3(s_1)]$$
$$+\frac{\partial p_1(s_1,h)}{\partial h}\left[\phi'(h) - \phi'(h)(1-p_3(s_1) - \frac{dp_3(s_1)}{ds_1}\frac{1}{d\sigma(s_1)}\right]$$

Note that we can cancel terms using an expression analogous to 23 in the main text; when the Challenger separates, then the following must be true

$$\frac{dp_3(s_1)}{ds_1}\frac{1}{d\sigma(s_1)}p_1(s_1,h)+\left[\frac{\partial p_1(s_1,h)}{\partial s_1}\frac{1}{d\sigma(s_1)}+\frac{\partial p_1(s_1,h)}{\partial h}\right]p_3(s_1) \tag{B. 12}$$

Applying that identity to $\phi''(h)$ and then substituting what remains into the second order condition, we find that we are left with

$$\phi'(h)\frac{\partial p_1(s_1,h)}{\partial h}p_3(s_1)-\frac{\partial^2 p_1(s_1,h)}{\partial s_1\partial h}\frac{1}{d\sigma(s_1)}[\phi(h)+(1-\phi(h)(1-p_3(s_1))]$$
$$+\frac{\partial p_1(s_1,h)}{\partial h}\frac{dp_3(s_1)}{ds_1}\frac{1}{d\sigma(s_1)}[1-\phi(h)]<0$$

Note that we can use B. 12 to substitute in for the partial derivative in the first term, rearrange, and divide by $d\sigma(s_1)$ to find that

$$\frac{\partial^2 p_1(s_1,h)}{\partial s_1\partial h}>\frac{\phi'(h)}{\phi(h)+(1-\phi(h)(1-p_3(s_1))}\left[\frac{\partial p_1(s_1,h)}{\partial s_1}p_3(s_1)+\frac{dp_3(s_1)}{ds_1}p_1(s_1,h)\right]$$
$$+\frac{\partial p_1(s_1,h)}{\partial h}\frac{dp_3(s_1)}{ds_1}\frac{1-\phi(h)}{\phi(h)+(1-\phi(h)(1-p_3(s_1))}$$

Substituting in for the hazard rate produces the inequality in equation B. 4.

*Step 2:* The Challenger's utility will satisfy single crossing whenever it is the case that

$$\phi(h)[p_1(s_1,h)-c_1^b]+[1-\phi(h)][p_1(s_1,h)(1-p_3(s_1)-c_1^t]\geq$$
$$\phi(h')[p_1(s_1,h')-c_1^b]+[1-\phi(h')][p_1(s_1,h')(1-p_3(s_1)-c_1^t]$$

where $h>h'$, and when

$$\phi(h')[p_1(s_1',h')-c_1^b]+[1-\phi(h')][p_1(s_1',h')(1-p_3(s_1')-c_1^t]\geq$$
$$\phi(h)[p_1(s_1',h)-c_1^b]+[1-\phi(h)][p_1(s_1',h)(1-p_3(s_1')-c_1^t]$$

where $s_1 > s_1'$. Adding the two inequalities together and rearranging, we find that

$$\phi(h)[p_1(s_1,h) - p_1(s_1',h)] + [1-\phi(h)][p_1(s_1,h)(p_3(s_1') - p_3(s_1)) + [1-p_3(s_1')][p_1(s_1,h) - p_1(s_1',h)]] \geq$$
$$\phi(h')[p_1(s_1,h') - p_1(s_1',h')] + [1-\phi(h')][p_1(s_1,h')(p_3(s_1') - p_3(s_1) - [1-p_3(s_1')][p_1(s_1,h') - p_1(s_1',h')]]$$

When we divide this expression by $\Delta s_1$, take the limit of $\Delta s_1$ to zero, and rearrange we find that

$$\begin{aligned} &[\phi(h) - \phi(h')]\frac{\partial p_1(s_1,h)}{\partial s_1} + \phi(h')\left[\frac{\partial p_1(s_1,h)}{\partial s_1} - \frac{\partial p_1(s_1,h')}{\partial h}\right] \\ &+[1-\phi(h)]\left[-[p_1(s_1,h) - p_1(s_1,h')]\frac{dp_3(s_1)}{ds_1} + [1-p_3(s_1)]\left(\frac{\partial p_1(s_1,h)}{\partial s_1} - \frac{\partial p_1(s_1,h')}{\partial s_1}\right)\right] \\ &-[\phi(h) - \phi(h')]\left[-p_1(s_1,h')\frac{dp_3(s_1)}{ds_1} + [1-p_3(s_1)]\frac{\partial p_1(s_1,h')}{\partial s_1}\right] \geq 0 \end{aligned}$$

Dividing by $\Delta h$, taking the limit of $\Delta h$ to zero, and rearranging, we are left with

$$\begin{aligned} &-\phi'(h)\left[p_3(s_1)\frac{\partial p_1(s_1,h)}{\partial s_1} + p_1(s_1,h)\frac{dp_3(s_1)}{ds_1}\right] \\ &+\frac{\partial^2 p_1(s_1,h)}{\partial s_1 \partial h}[\phi(h) + (1-\phi(h))(1-p_3(s_1))] - [1-\phi(h)]\frac{\partial p_1(s_1,h)}{\partial h}\frac{dp_3(s_1)}{ds_1} \geq 0 \end{aligned}$$

which is the same expression as the second order condition. ■

### B.4.2 Proof of Lemma B. 2

To prove the lemma, I provide a complete characterization of the equilibrium when equation 9 does not hold and prove its existence. This will show that even when types of the Challenger pool on the maximal demand, the third party will not intervene against them even when they do not signal. Moreover, many of the calculations performed in this lemma will be useful in proving Proposition B.1.

*Step 1: Characterizing the Equilibrium*: We will construct the Challenger's and Defender's signaling strategies and show that the third party's best response is not to intervene. First, the weakest types of the Challenger can separate by making a unique demand and will then have the third party intervene. These types will separate by making demands $x = p_1(s_1)(1 - p_3(s_1)) + c_2^t$ so that the Defender is indifferent between accepting and not. Separation in response to such demands continues to function much the same as it did in the main text. The Defender is indifferent in response to any such demand and selects a strategy designed to ensure that the separating types find their prescribed demand incentive compatible

$$\frac{-\psi'(x)}{\psi(x)} = \frac{1}{x - p_1(s_1, 0)(1 - p_3(s_1) + c_1^t}$$

which is an ordinary differential equation with a similar interpretation to equation 14 in the main text and with solution

$$\psi(x) = e^{-\frac{x - p(\underline{s}_1)(1 - p_3(\underline{s}_1)) - c_2^t}{c_1^t + c_2^t}}$$

There are then two possible cases to consider. First, it is possible that $\widetilde{s}_1^3 < \widetilde{s}_1^b$. In this case, types weaker than $\widetilde{s}_1^3$ play the strategy described above, and types stronger than $\widetilde{s}_1^3$ can continue to separate by demanding $x = p_1(s_1, 0) + c_2^b$ which keeps the Defender indifferent. The resulting discontinuity in the Challenger's bargaining strategy generates a mirroring discontinuity in the Defender's concession strategy. Specifically the Defender plays

$$\psi(p_1(\widetilde{s}_1, 0) + c_2^b) = \frac{e^{-\frac{x - p(\underline{s}_1)(1 - p_3(\underline{s}_1)) - c_2^t}{c_1^t + c_2^t}}[c_1^t + c_2^t] - (c_1^t - c_1^b) - p_1(\widetilde{s}_1^3, 0)p_3(\widetilde{s}_1^3)}{c_1^b + c_2^b}$$

which ensures that type $\widetilde{s}_1^3$ is indifferent between demanding $x = p_1(\widetilde{s}_1^3), 0)(1 - p_3(\widetilde{s}_1^3))$ and fighting both the Defender and the third party and demanding $x = p_1(\widetilde{s}_1^3, 0) + c_2^b$ and fighting the Defender alone. After this the Defender's rate of concession simply decreases at a steady rate

$$\frac{-\psi'(x)}{\psi(x)} = \frac{1}{x - p_1(s_1, 0) + c_1^b}$$

for demands in the range $[p_1(\widetilde{s}_1^3, 0) + c_2^b, p_1(\rho) + c_2^b]$, which continues to ensure that the Challenger is willing to separate. Finally, as in the main text there can be an additional discontinuity as types in the range $[\rho, \bar{s}_1]$ pool and demand $x = 1$ where type $\rho$ is defined analogously to equation 3 in the main text.[8]

Second, it is possible that it is the case that the $\widetilde{s}_1^3 > \widetilde{s}_1^b$. In this case types in the range $[\underline{s}_1, \rho)$ Challenger separate by demanding $x = p_1(s_1)(1 - p_3(s_1)) + c_2^t$ and types in the range $[\rho, \bar{s}_1]$ demand $x = 1$ where $\rho$ is defined as in 3 in the main text.[9]. In both cases the types pooling on demand $x = 1$ form a connected interval of all types stronger than $\rho$. If the inequality in B. 6 does not hold, then third party will strictly prefer not to intervene in the conflict.

*Step 2: The Challenger's Expected Utility Function is Concave:* Absent handicapping the Challengers expected utility in equation B. 1 can be simplified to the following whenever the Challenger expects the third party to intervene

$$x\psi(x) + [1 - \psi(x)][p_1(s_1, 0)(1 - p_3(s_1)) - c_1^t] \tag{B. 13}$$

Taking the second derivative of the Challenger's expected utility function with respect to $x$ demonstrates that the function will be concave if

$$2\psi'(x) + \psi''(x)[x - p_1(s_1, 0)(1 - p_3(s_1)) + c_1^t]$$

[8]As in the main text, this generates an additional discontinuity in the Defender's strategy where the Defender selects a concession rate $\psi(1)$ such that type $\rho$ is indifferent between demanding $x = 1$ and demanding $x = p_1(\rho, 0) + c_2^b$.

[9]See footnote 8 above.

Recalling that the Defender's strategy must satisfy

$$\frac{-\psi'(x)}{\psi(x)} = \frac{1}{c_1^t + c_2^t}$$

we can rearrange and take derivative of this function with respect to $x$ to find that

$$\psi''(x) = -\psi'(x)(c_1^t + c_2^t)$$

which when we substitute back into the second order condition, we are left with

$$\psi'(x) < 0$$

which is true given that $\psi(x)$ is a strictly decreasing function.

The proof that the Challenger's expected utility is concave in $x$ when the Challenger does not expect the third party to intervene is identical to the proof of Proposition 1 in the main text.

*Step 3: The Challenger's Expected Utility Function Satisfies Single Crossing:* All that remains is to demonstrate that there are no profitable deviations from or to the "corner solution" $x = 1$. Taking the cross partial of the Challenger's utility in B. 13 with respect to $x$ and $s_1$ and seeing that it produces

$$-\psi'(x)\left[\frac{\partial p_1(s_1,0)}{\partial s_1}(1 - p_3(s_1)) - \frac{dp_3(s_1)}{ds_1}p_1(s_1,0)\right]$$

which is positive. Along with footnote 11 in the main text establishing that the Challenger's expected utility function satisfies single crossing when the Challenger can expect a bilateral war, we can establish single crossing for any $x$. This rules out any deviation by any type weaker than $\rho$ to $x = 1$ and any type stronger than $\rho$ to a demand less than 1. ■

### B.4.3 Proof of Proposition B. 1

Lemma B. 1 shows that the Challenger's handicapping strategy is incentive compatible. The proof that the Challenger's bargaining strategy is incentive compatible follows from the concavity and single crossing proofs in lemma B. 2. The Defender is indifferent in response to any given demand by construction and the third party is indifferent in response to any positive level of handicapping. It follows that the strategies described in the proposition constitute an equilibrium. ■

# C Handicap Signaling with Two-sided Incomplete Information

I analyze the effect of two-sided incomplete information on handicap signaling and show that a semi-separating equilibrium remains viable. Though key qualitative features of the equilibrium under asymmetric information are preserved in this new environment, the players' strategies require adjustments. Specifically, the Challenger can no longer aim to make the Defender indifferent because it is unsure of the Defender's type. As a result, the Defender plays a cutoff strategy and either chooses to fight or back down. When the Challenger signals, it sets the risk of a decreased wartime-payoff if the Defender is strong against the benefit of deterring a potentially weak Defender. Because stronger types incur smaller penalties for higher handicaps, they remain more willing to adopt higher handicaps and are also more likely to deter the Defender. However, it is no longer the case that all the types of the Challenger who engage in signaling handicap themselves to the same probability of winning. Moreover, the existence of a semi-separating equilibrium and the strategies adopted by the Challenger become contingent on the distribution of the Defender's types.

## C.1 Model Setup

Suppose that the good is once more indivisible so that the structure of the game is such that the Challenger signals, the Defender observes that signal, and then the Defender decides whether or not to fight. The wartime outcome is now determined by the lottery function $p_1(s_1, s_2, h)$ where $p_1(\cdot,\cdot,\cdot)$ is strictly increasing in $s_1$ and strictly decreasing in $s_2$ and $h$. The strength parameters $s_1, s_2$ are private information where $s_1$ is distributed as before and $s_2$ is distributed according to the continuous, strictly increasing, and common knowledge cumulative distribution function $F_2 = [\underline{s_2}, \bar{s_2}]$. The Defender's strategy is to select a probability with which to back down as a function of both its type and the observed signal so that $\phi : s_2 \times h \rightarrow [0, 1]$. I assume that there exist types of the Challenger that the weakest types of the Defender would prefer not to fight. That is, there exist types $s_1$ such that

$$p_2(s_1, \underline{s_2}, 0) \leq c_2 \tag{C. 1}$$

Let $\widetilde{s_1}$ denote the type of $s_1$ for which equation C. 1 holds with equality. Finally, Assumption 2 guarantees that stronger types of the Challenger continue to incur smaller penalties for handicapping themselves.

**Assumption C. 1**

$$p_1(s_1', s_2, h') - p_1(s_1', s_2, h) > p_1(s_1, s_2, h') - p_1(s_1, s_2, h)$$
$$\forall\ h' < h \in [0, \bar{h}]\ \textit{and}\ \forall\ s_2 \in [\underline{s_2}, \bar{s_2}]\ \textit{if and only if}\ s_1 > s_1'$$

Note, that I have not assumed any relationship between the Defender's strength and the size of the handicap on the countries' probability of winning.

## C.2 Equilibrium Characterization

If handicap signals continue to be informative, then the Defender's best response to a handicap signal is to stand firm if its expected utility from fighting the handicapped signaler is higher than its payoff from backing down. This remains true if separation fails locally. Define $\overline{\sigma}^{-1}(h)$ as the strongest type of the Challenger choosing to signal with $h$ and $\underline{\sigma}^{-1}(h)$ as the weakest type of the Challenger choosing to signal with $h$. Then the Defender's pure strategy is given by

$$\phi^*(s_2, h) \equiv \begin{cases} 1 \text{ if } & p_2(\sigma^{-1}(h), s_2, h) \leq c_2 \quad \text{and } \overline{\sigma}^{-1}(h) = \underline{\sigma}^{-1}(h) \\ 1 \text{ if } & \int\limits_{\{\sigma^{-1}(h)\} \in S_1} p_2(s_1, s_2, h) f(s_1) ds_1 \leq c_2 \quad \text{and } \overline{\sigma}^{-1}(h) \neq \underline{\sigma}^{-1}(h) \\ 0 \text{ if } & p_2(\sigma^{-1})(h), s_2, h) > c_2 \quad \text{and } \overline{\sigma}^{-1}(h) = \underline{\sigma}^{-1}(h) \\ 0 \text{ if } & \int\limits_{\{\sigma^{-1}(h)\} \in S_1} p_2(s_1, s_2, h) f(s_1) ds_1 > c_2 \quad \text{and } \overline{\sigma}^{-1}(h) \neq \underline{\sigma}^{-1}(h) \end{cases} \tag{C. 2}$$

This implies that the Defender is playing a cutoff strategy. In this case the Challenger's payoff becomes

$$U_1(s_1, h) = F(\widetilde{s_2}(h)) + (1 - F(\widetilde{s_2}(h))) \left[ \frac{\int_{\widetilde{s_2}(h)}^{\bar{s_2}} p_1(s_1, s_2, h) f(s_2)}{1 - F(\widetilde{s_2}(h))} - c_1 \right] \tag{C. 3}$$

where $\widetilde{s_2}(h)$ is the threshold type of the Defender who is indifferent between fighting types of the Challenger who signal with $h$ and backing down. The first term is the payoff that the Challenger receives from signaling and having types of the Defender below the threshold type back down. The second term is the expected value of fighting those types of the Defender which choose to stand firm.

It remains to show when and how the Challenger can separate. The following Lemma provides conditions under which the Challenger's utility function satisfies the single crossing condition so that its strategy is weakly increasing in its type $s_1$. This in turn implies that any set of types choosing to adopt the same signal must form a connected interval $[\underline{\sigma}^{-1}(h), \overline{\sigma}^{-1}(h)]$.[10]

**Lemma C. 1**

*The Challenger's utility function in equation C. 3 satisfies single-crossing if*

$$\int_{\widetilde{s_2}(h)}^{\overline{s_2}} \frac{\partial^2 p_1(s_1, s_2, h)}{\partial s_1 \partial h} f(s_2) ds_2 \geq \frac{\frac{\partial p_1(s_1, \widetilde{s}_2(h), h)}{\partial s_1}}{\frac{\partial p_2(s_1, \widetilde{s}_2(h), h)}{\partial h}} f(\widetilde{s}_2(h)) \tag{C. 4}$$

All proofs are presented at the end of this section. Regardless of whether the Challenger plays a strategy in which types separate or pool, so long as this condition holds, stronger types will be willing to adopt higher handicaps and will be more likely to deter. This condition is easily satisfied. Per Assumption 2, the left-hand side of equation C. 4 is positive.

Two more requirements are necessary for there to exist a semi-separating equilibrium where all signaling types separate. First, there must be a sufficient mass of types to be deterred so that the Challenger will be willing to risk fighting the strongest types of the Defender with a handicap. The weakest signaling type will be the weakest type for which such a handicap exists. Formally, let $\widehat{s}_1$ be the weakest such type as defined by the following equation

$$\begin{aligned} \widehat{s}_1 \equiv \min \Bigg\{ s_1 \text{ for which } \exists\ h : \int_{\underline{s}_2}^{\bar{s}_2} p_1(s_1, s_2, 0) f_2(s_2) ds_2 - c_1 = \\ F(\widetilde{s}_2(h)) + (1 - F(\widetilde{s}_2(h))) \left[ \frac{\int_{\widetilde{s}_2(h)}^{\bar{s}_2} p_1(s_1, s_2, h) f(s_2)}{1 - F(\widetilde{s}_2(h))} - c_1 \right] \Bigg\} \end{aligned} \tag{C. 5}$$

Second, each type stronger than $\widehat{s}_1$ must be able to find a handicap that leaves it indifferent between separating and not. Formally, let $H(s_1)$ denote the supremum of the set of signals used by types smaller than $s_1$. Separation requires that $s_1$ separate by adopting a handicap $h \geq H(s_1)$ such that $U_1(s_1, h) = U_1(s_1, H(s_1))$. The single crossing condition then guarantees

[10] This is useful in proving the existence of a semi-separating equilibrium because it implies that when considering deviations it is only necessary to consider those employed by the "nearest types" rather than the universe of possible deviations.

that weaker types strictly prefer $H(s_1)$ over $h$.

Whether or not these conditions holds depends on the distribution of the Defender's type. This is made clear and summarized in the following Proposition.

**Proposition C. 1**

*A semi separating equilibrium in which all signaling types separate and the Defender plays according to equation C. 2 requires that:*

*(i) Equation C. 4 be satisfied*

*(ii) There exist a type $\widehat{s}_1$ that plays a signal $h$ satisfying equation C. 5*

*(iii) For for each type $s_1 > \widehat{s}_1$ there exist a handicap $h \geq H(s_1)$ that satisfies*

$$f(\widetilde{s}_2(H(s_1))) = \frac{-\int_{\widetilde{s}_2(H(s_1))}^{\overline{s_2}} \frac{\partial p_1(s_1,s_2,h) f(s_2) ds_2}{\partial h}}{[1 + c_1 - p_1(s_1, \widetilde{s}_2, h)] \frac{\partial p_2(s_1,\widetilde{s}_2(h),h)}{\partial h}} \tag{C. 6}$$

*or*

$$F(\widetilde{s}_2(h)) - F(\widetilde{s}_2(H(s_1))) = \frac{\int_{\widetilde{s}_2(h)}^{\bar{s_2}} p_1(s_1, s_2, h) f(s_2) ds_2 - \int_{\widetilde{s}_2(H(s_1))}^{\bar{s_2}} p_1(s_1, s_2, H(s_1)) f(s_2) ds_2}{1 + c_1} \tag{C. 7}$$

*Types $s_1 < \widetilde{s}_1$ play $h = 0$.*

Note that Proposition C. 1 only deals with a semi-separating equilibrium where all types perfectly separate. If the conditions required for full separation fail, handicap signaling may still be viable with a semi-separating equilibrium where connected intervals of the Challenger's type pool on a certain demand. The conditions required for the existence of such an equilibrium are similar to those required for full separation: single-crossing, a sufficient mass of weak types of the Defender that can be deterred, and that the weakest and strongest types pooling on a given signal be indifferent between the signal they use and equilibrium in those used by types in their immediate neighborhood.

## C.3 Section Proofs

### C.3.1 Proof of Lemma C. 1

This proof follows similar steps to those in Lemma 2 in the main text. In an equilibrium with $\sigma(s_1) = h > \sigma(s_1') = h'$ for $s_1 > s_1'$ the following equations must hold

$$\begin{gathered} F_2(\widetilde{s_2}(h)) + (1 - F_2(\widetilde{s_2}(h))) \left[ \frac{\int_{\widetilde{s_2}(h)}^{\bar{s_2}} p_1(s_1, s_2, h) f(s_2) ds_2}{1 - F_2(\widetilde{s_2}(h))} - c_1 \right] \geq \\ F_2(\widetilde{s_2}(h')) + (1 - F_2(\widetilde{s_2}(h'))) \left[ \frac{\int_{\widetilde{s_2}(h')}^{\bar{s_2}} p_1(s_1, s_2, h') f(s_2) ds_2}{1 - F_2(\widetilde{s_2}(h'))} - c_1 \right] \end{gathered} \tag{C. 8}$$

and

$$\begin{gathered} F_2(\widetilde{s_2}(h')) + (1 - F_2(\widetilde{s_2}(h'))) \left[ \frac{\int_{\widetilde{s_2}(h')}^{\bar{s_2}} p_1(s_1', s_2, h') f(s_2) ds_2}{1 - F_2(\widetilde{s_2}(h'))} - c_1 \right] \geq \\ F_2(\widetilde{s_2}(h)) + (1 - F_2(\widetilde{s_2}(h))) \left[ \frac{\int_{\widetilde{s_2}(h)}^{\bar{s_2}} p_1(s_1', s_2, h) f(s_2) ds_2}{1 - F_2(\widetilde{s_2}(h))} - c_1 \right] \end{gathered} \tag{C. 9}$$

Adding the inequalities given by C. 9 and C. 8 and rearranging we obtain

$$\begin{aligned} &\int_{\widetilde{s_2}(h)}^{\bar{s_2}} p_1(s_1, s_2, h) f(s_2) ds_2 - \int_{\widetilde{s_2}(h)}^{\bar{s_2}} p_1(s_1', s_2, h) f(s_2) ds_2 \\ \geq &\int_{\widetilde{s_2}(h')}^{\bar{s_2}} p_1(s_1, s_2, h') f(s_2) ds_2 - \int_{\widetilde{s_2}(h')}^{\bar{s_2}} p_1(s_1', s_2, h') f(s_2) ds_2 \end{aligned}$$

Dividing both sides by the difference between $\Delta s_1$

$$\frac{\int_{\widetilde{s_2}(h)}^{\bar{s_2}} [p_1(s_1, s_2, h) - p_1(s_1', s_2, h)] f(s_2) ds_2}{\Delta s_1} \geq \frac{\int_{\widetilde{s_2}(h')}^{\bar{s_2}} [p_1(s_1, s_2, h') - p_1(s_1', s_2, h')] f(s_2) ds_2}{\Delta s_1}$$

Taking the limit as $\Delta s_1$ goes to 0, rearranging, and then dividing by $\Delta h$

$$\int_{\widetilde{s_2}(h)}^{\bar{s_2}} \frac{\left[ \frac{\partial p_1(s_1, s_2, h)}{\partial s_1} - \frac{\partial p_1(s_1, s_2, h')}{\partial s_1} \right] f(s_2) ds_2}{\Delta h} - \int_{\widetilde{s_2}(h')}^{\widetilde{s_2}(h)} \frac{\frac{\partial p_1(s_1, s_2, h')}{\partial s_1} f(s_2) ds_2}{\Delta h} \geq 0$$

However, taking the limit as $\Delta h$ goes to 0 is not as straightforward as in Lemma 2. While the limit can transfer through the first integral, the second variable has the limit of the integral go to zero as the bounds of integration approach each other while also having the denominator go to zero. Therefore, we must use L'Hôpital's rule and take the derivative of the second term. Taking these steps, we find that as per Liebniz's rule we are left with

$$\int_{\widetilde{s_2}(h)}^{\overline{s_2}} \frac{\partial^2 p_1(s_1, s_2, h)}{\partial s_1 \partial h} f_2(s_2) ds_2 - \left[ \lim_{h' \to h} \frac{-\frac{\partial p_1(s_1, \widetilde{s}_2, h)}{\partial s_1} f_2(\widetilde{s}_2) \cdot \frac{d\widetilde{s}_2(h)}{dh} + \int_{\widetilde{s}_2(h')}^{\widetilde{s}_2(h)} \frac{\partial^2 p_1(s_1, s_2, h)}{\partial s_1 \partial h} f_2(s_2) ds_2}{-1} \right]$$

Note that the second term in the numerator in the square brackets goes to zero when the limit is taken. Moreover, recall that $\widehat{s}_2(h)$ is the inverse function $s_2(h) = p_2^{-1}(s_1, c_2, h)$. Therefore, we are left

$$\int_{\widetilde{s_2}(h)}^{\overline{s_2}} \frac{\partial^2 p_1(s_1, s_2, h)}{\partial s_1 \partial h} f_2(s_2) ds_2 - \frac{\frac{\partial p_1(s_1, \widetilde{s}_2(h), h)}{\partial s_1}}{\frac{\partial p_2(s_1, \widetilde{s}_2(h), h)}{\partial h}} f(\widetilde{s}_2(h)) \geq 0$$

This implies that the single crossing condition is satisfied whenever equation C. 4 holds. ■

### C.3.2 Proof of Proposition C. 1

The proof of this proposition begins by demonstrating that every type adopting a positive handicap $h > 0$ in a separating equilibrium must always be indifferent between separating and not so that $U_1(s_1, h) = \lim_{h' \to H(s_1)} U_1(s_1, h')$ for $h' < h$. To see why suppose that it is the case that $U_1(s_1, h) < \lim_{h' \to H(s_1)} U_1(s_1, h')$ for all $h \geq H(s_1)$ in which case the Challenger would choose not to separate. If, on the other hand, it was the case that $U_1(s_1, h) > \lim_{h' \to H(s_1)} U_1(s_1, h')$, then $s_1$ will separate and increases their payoff by some $\epsilon > 0$. Next observe that

$$U(s_1, h) = F(\tilde{s_2}(h))(1 + c_1) + \int_{\widetilde{s_2}(h)}^{\overline{s_2}} p_1(s_1, s_2, h) f_2(s_2) ds_2 - c_1$$

implying that $U(s_1, h)$ is continuous in $s_1$ (Ramsay and Fey 2011, Proposition 1). This implies that there exists a type $s_1' < s_1$ for whom $U(s_1', h) = \lim_{h' \to H(s_1)} U_1(s_1, h')$. Single-crossing would then imply that all types in the interval $(s_1, s_1')$ strictly prefer to deviate to the signal $h$. This contradicts the premise that the signal is separating. Therefore if $s_1$ separates it must be the case that $U(s_1, h) = \lim_{h' \to H(s_1)} U_1(s_1, h')$.

I now proceed to show that the above implies that separation must be given by the strategies laid out in the proposition. First, note that the requirement the weakest signaling type be indifferent between signaling and not implies that there exist a weakest signaling type $\widehat{s}_1$ as defined in equation C. 5. The single-crossing condition then implies that types weaker than $\widehat{s}_1$ strictly prefer not to signal. Types stronger than $s_1$ must similarly be indifferent between separating and not. That is they must find a handicap $h \geq H(s_1)$ such that the following holds in the limit for $h' < H(s_1)$

$$\lim_{h' \to H(s_1)} F_2(\widetilde{s_2}(h)) + (1 - F_2(\widetilde{s_2}(h))) \left[ \frac{\int_{\widetilde{s_2}(h')}^{\bar{s_2}} p_1(s_1, s_2, h) f(s_2)}{1 - F_2(\widetilde{s_2}(h))} - c_1 \right] =$$
$$F_2(\widetilde{s_2}(h')) + (1 - F_2(\widetilde{s_2}(h'))) \left[ \frac{\int_{\widetilde{s_2}(h')}^{\bar{s_2}} p_1(s_1, s_2, h') f(s_2)}{1 - F_2(\widetilde{s_2}(h'))} - c_1 \right]$$

If $h \neq H(s_1)$, it is straightforward to derive C. 7 by taking the limit and rearranging. However, if $h = H(s_1)$, then first rearrange so that the equation above becomes

$$[F_2(\widetilde{s_2}(h)) - F_2(\widetilde{s_2}(h'))](1 + c_1) = \int_{\widetilde{s_2}(h')}^{\bar{s_2}} p_1(s_1, s_2, h) f(s_2)$$
$$- p_1(s_1, s_2, h) f(s_2) + \int_{\widetilde{s_2}(h')}^{\widetilde{s_2}(h)} p_1(s_1, s_2, h') f(s_2)$$

Then, dividing both sides by $\Delta h$ and taking the limit as $h' \to h$ gives equation C. 6.

Finally, I proceed to rule out deviations from the Challenger's prescribed strategy. First, note that handicap signaling is costly. It follows that no type will ever seek to adopt an off-path handicap signal higher than that necessary to achieve separation. The single-crossing property guarantees that when types perfectly separate by playing an $h$ for which C. 7 or C. 6 holds so that stronger types adopt higher handicap signals, no type of the Challenger can increase its payoff by adopting the handicap adopted by a weaker or stronger type. This implies that the postulated strategy is expected utility maximizing. ■

# D The Guns-Butter Trade-off and Handicaps

Recall that when China invaded Vietnam in 1979, the latter choose to keep troops in Cambodia and continued to reap the benefits of that conquest. This example illustrates that if handicaps take the form-of under-investment in military strength, then countries may benefit from consumption of those resources elsewhere (Powell 1993). If the handicap generates some welfare that is independent of the good being fought over, then handicapping may no longer be costly and consequently no longer an effective signal.

The following demonstrates that handicaps continue to convey information so long as the "butter" benefit is not too large. Let $u : h \rightarrow \mathbb{R}^1$ be a strictly increasing and bounded function capturing the gains the consumption benefits derived from under-investment in military strength. There are four possible ways in which the benefit may be distributed: the benefit is received by the Challenger only if war is avoided, the Challenger receives the benefit so long as it receives the good, the Challenger receives the benefit independently of the outcome, or the Challenger augments the value of the good being bargained over so that both the Challenger and the Defender might get $u(h)$ if they get the good. I explore each of these cases here and demonstrate that handicaps can remain informative signals in all of them so long as the benefit does not overwhelm the cost of handicapping.

## D.1 Generating Contingent Value

If handicaps generate value that is contingent on war not taking place, then the Challenger's expected utility function becomes

$$\phi(h)(1+u(h)) + (1-\phi(h))[p_1(s_1,h) - c_1] \tag{D. 1}$$

In this instance, the Defender is still indifferent when $p_2(s_1,h) = c_2$. This implies that the Challenger can continue to separate by playing the same strategy $\sigma_h^*$ as in equation 11 in the main text. However, the Defender's strategy is designed to ensure that the Challenger finds $\sigma^*$ incentive compatible and must change with the introduction of $u(h)$. Now the Challenger will seek to handicap itself up until the point where the following equation holds

$$\frac{\phi'(h)}{1-\phi(h)} = -\frac{\frac{\partial p_1(s_1,h)}{\partial h}}{c_1+c_2+u(h)} - \frac{\phi(h)}{1-\phi(h)}\frac{u'(h)}{c_1+c_2+u(h)} \tag{D. 2}$$

The left-hand side is the marginal increase in the probability that the Defender concedes if the Challenger increases their handicap. The right-hand side is the marginal cost of handicapping minus the marginal increase in independent benefit gained if the Defender backs down multiplied by the probability of receiving that benefit. Both of the terms on the right-hand side are weighted by the difference in payoffs from having the Defender concede as opposed to going to war. Unlike the hazard rate given by equation 14 in the main text, the hazard rate in the equation above does not have a closed form solution.

The following proposition provides a characterization of the equilibrium. It shows that the independent benefit makes the sufficient condition on the cross partial more restrictive. Therefore, it is more difficult to achieve an equilibrium with signaling.

**Proposition D. 1**

*If the cross-partial on the lottery function satisfies*

$$\frac{\partial^2 p_1(s_1,h)}{\partial s_1 \partial h} > -\frac{\frac{\partial p_1(s_1,h)}{\partial s_1}\frac{\partial p_1(s_1,h)}{\partial h}}{c_1+c_2+u(h)} - \frac{u'(h)\frac{\partial p_1(s_1,h)}{\partial h}}{c_1+c_2+u(h)}\frac{\phi(h)}{1-\phi(h)} \tag{D. 3}$$

*then*

(i) *There exists a Perfect Bayesian Equilibrium where the Challenger plays $\sigma_h^*$ as in equation 11 in the main text and the Defender responds by playing equation D. 2 with $\phi(0)=0$.*

(ii) *The uniqueness arguments in Proposition 2 parts (ii) and (iii) hold.*

**Proof**: The proof proceeds as it did for Proposition 2 in the main text: I show that the condition D. 3 is necessary for the Challenger's expected utility function to be concave in $h$ and that this is sufficient for the Challenger's expected utility function to satisfy the single crossing property. The remaining arguments are identical to those in the main text.

As before, D. 1 will be concave in $h$ whenever its second derivative with respect to $h$ is negative

$$\begin{aligned}\phi''(h)(u(h)+1)+2u'(h)\phi'(h)+\phi(h)u''(h)+(1-\phi(h))\frac{\partial^2 p_1(s_1,h)}{\partial^2 h}\\ -2\phi'(h)\frac{\partial p_1(s_1,h)}{\partial h}-\phi''(h)(p_1(s_1,h)-c_1)<0\end{aligned}$$

Again, there can be no progress without first learning more about $\phi''(h)$. Therefore, we take the derivative of $\phi'(h)$ as given in D. 2 with respect to $h$ while noting that in D. 2 $s_1$ is given by $\sigma^{-1}(h)$. This leaves us with

$$\begin{aligned}\phi''(h)(u(h)+c_1+c_2)-\phi'(h)\left[\frac{\partial p_1(s_1,h)}{\partial s_1}\frac{1}{d\sigma(s_1)}+\frac{\partial p_1(s_1,h)}{\partial h}\right]+\phi'(h)u'(h)=\\ -\frac{\partial^2 p_1(s_1,h)}{\partial s_1 \partial h}(1-\phi(h))\frac{1}{\frac{d\sigma_1(s_1)}{ds_1}}-\frac{\partial^2 p_1(s_1,h)}{\partial^2 h}(1-\phi(h))-\phi'(h)u'(h)-u''(h)\phi(h)\end{aligned}$$

We can apply 23 to cancel out terms and then substitute this expression into the second order condition and rearrange to find that we are left with

$$\frac{1}{d\sigma(s_1)}\frac{\partial^2 p_1(s_1,h)}{\partial s_1 \partial h} > \frac{\phi'(h)}{1-\phi(h)}\frac{\partial p_1(s_1,h)}{\partial h}$$

Adding and subtracting

$$\frac{\partial p_1(s_1,h)}{\partial s_1}\frac{1}{d\sigma(s_1)} \cdot \frac{\phi'(h)}{1-\phi(h)}$$

to the right-hand side of the second-order condition, applying 23, multiplying terms by $d\sigma(s_1)$, and then substituting in for D. 2, we find the expression in equation D. 3.

We can now show that the single crossing condition will be satisfied whenever the inequality in equation D. 3 holds. We begin by examining the incentive compatibility constraints which are now given by

$$\phi(h)(1+u(h)) + (1-\phi(h))(p_1(s_1,h)-c_1) \geq \phi(h')(1+u(h')) + (1-\phi(h'))(p_1(s_1,h')-c_1)$$
$$\phi(h')(1+u(h')) + (1-\phi(h'))(p_1(s_1',h')-c_1) \geq \phi(h)(1+u(h)) + (1-\phi(h))(p_1(s_1',h)-c_1)$$

where $s_1 > s_1'$ and $h > h'$. Adding the inequalities and following analogous steps to those in Proposition 2 of the main text, we find that the incentive compatibility constraints hold if

$$\frac{\partial^2 p_1(s_1,h)}{\partial s_1 \partial h} \geq \frac{\phi'(h)}{1-\phi(h)}\frac{p_1(s_1,h)}{\partial s_1}$$

which is identical to the expression in D. 3 after substituting for the hazard rate.

The same arguments that established that this is sufficient to guarantee the existence of a Perfect Bayesian Equilibrium along with those establishing uniqueness, including those involving D1, are the same as those used in Proposition 2 in the main text. ■

## D.2 Augmenting the Value of the Good for the Challenger

If the Challenger enjoys the benefits of handicapping only if they receiver the good, regardless of whether through concession or through war, then the Challenger's expected utility function becomes

$$\phi(h) + (1 - \phi(h))(p_1(s_1, h) - c_1) + u(h)[\phi(h) + (1 - \phi(h))p_1(s_1, h)] \tag{D. 4}$$

In this instance, the Defender is still indifferent when $p_2(s_1, h) = c_2$. This implies that the Challenger can once again continue to separate by playing the same strategy $\sigma_h^*$ as in equation 11 in the main text. However, the Defender's strategy must once again change to ensure that the Challenger will find $\sigma^*$ incentive compatible. Now the Challenger will seek to handicap itself up until the point where the following equation holds

$$\frac{\phi'(h)}{1 - \phi(h)} = -\frac{\partial p_1(s_1, h)}{\partial h}\left[\frac{1 + u(h)}{c_1 + c_2(1 + u(h))}\right] - \frac{u'(h)}{c_1 + c_2(1 + u(h))}\left[\frac{\phi(h)}{1 - \phi(h)} + p_1(s_1, h)\right] \tag{D. 5}$$

This equation has a similar interpretation to equation D. 2 with several minor differences. First, the first term on the right-hand side which captures the cost of handicapping is now equal to the marginal decrease in the probability of winning times the augmented value of victory $[1 + u(h)]$. Second, the second term on the right-hand side capturing the marginal independent benefit gained from handicapping is now multiplied by the probability that the rival concedes times the probability and the probability that the Challenger wins the war. Once again, this hazard rate does not have a closed form solution.

The following proposition provides a characterization of the equilibrium. It shows that the independent benefits, even when also tied to a successful wartime outcome, continue to make the sufficient condition on the cross partial more restrictive and an equilibrium with signaling more difficult to achieve.

**Proposition D. 2**

*If the cross partial on the lottery function satisfies*

$$\frac{\partial^2 p_1(s_1,h)}{\partial s_1 \partial h} > -\frac{\partial p_1(s_1,h)}{\partial s_1}\frac{\partial p_1(s_1,h)}{\partial h}\left[\frac{1+u(h)}{c_1+c_2(1+u(h))}\right] - \frac{u'(h)}{1+u(h)}\frac{\partial p_1(s_1,h)}{\partial s_1} - \frac{u'(h)\frac{\partial p_1(s_1,h)}{\partial s_1}}{c_1+c_2(1+u(h))}\left[\frac{\phi(h)}{1-\phi(h)}+p_1(s_1,h)\right] \tag{D. 6}$$

*then*

*(i) there exists a Perfect Bayesian Equilibrium where the Challenger plays $\sigma_h^*$ as in equation 11 in the main text and the Defender responds by playing equation D. 5 with $\phi(0) = 0$.*

*(ii) The uniqueness arguments in Proposition 2 parts (ii) and (iii) hold.*

**Proof**: The proof proceeds as it did for Proposition 2 in the main text: I show that condition D. 6 is necessary for the Challenger's expected utility function to be concave in $h$ and that this is sufficient for the Challenger's expected utility function to achieve single crossing.

As before, D. 4 will be concave in $h$ whenever its second derivative with respect to $h$ is negative

$$\begin{aligned}\phi''(h)[c_1+c_2(1+u(h))] - \phi'(h)\frac{\partial p_1(s_1,h)}{\partial h} + \phi'(h)u'(h)[1-p_1(s_1,h)] - \phi'(h)\frac{\partial p_1(s_1,h)}{\partial h}u(h)\\ +\frac{\partial^2 p_1(s_1,h)}{\partial^2 h}[1+u(h)][1-\phi(h)] + \frac{\partial p_1(s_1,h)}{\partial h}u'(h) - \phi'(h)\frac{\partial p_1(s_1,h)}{\partial h}[1+u(h)]\\ +u''(h)[\phi(h)+(1-\phi(h))p_1(s_1,h)] + u'(h)\phi'(h) - u'(h)\phi'(h)p_1(s_1,h)\\ +u'(h)\frac{\partial p_1(s_1,h)}{\partial h}[1-\phi(h)] < 0\end{aligned}$$

Again, there can be no progress without first learning more about $\phi''(h)$. Therefore, we take the derivative of $\phi'(h)$ as given in D. 5 with respect to $h$ while noting that in D. 5 $s_1$ is given by $\sigma^{-1}(h)$. This leaves us with

$$\begin{aligned}
\phi''(h)[c_1 + c_2(1+u(h))] - \phi'(h)\left[\frac{\partial p_1(s_1,h)}{\partial s_1}\frac{1}{d\sigma(s_1)} + \frac{\partial p_1(s_1,h)}{\partial h}\right] \\
+\phi'(h)u'(h)[1-p_1(s_1,h)] - \phi'(h)u(h)\left[\frac{\partial p_1(s_1,h)}{\partial s_1}\frac{1}{d\sigma(s_1)} + \frac{\partial p_1(s_1,h)}{\partial h}\right] \\
= -\frac{\partial^2 p_1(s_1,h)}{\partial s_1 \partial h}\frac{1}{d\sigma(s_1)}[1+u(h)][1-\phi(h)] - \frac{\partial^2 p_1(s_1,h)}{\partial^2 h}[1+u(h)][1-\phi(h)] \\
-\frac{\partial p_1(s_1,h)}{\partial h}u'(h) + \phi'(h)\frac{\partial p_1(s_1,h)}{\partial h}[1+u(h)] - u''(h)[\phi(h) + (1-\phi(h))p_1(s_1,h)] \\
-u'(h)\phi'(h) + u'(h)\phi'(h)p_1(s_1,h) - u'(h)\partial h[1-\phi(h)]\left[\frac{\partial p_1(s_1,h)}{\partial s_1}\frac{1}{d\sigma(s_1)} + \frac{\partial p_1(s_1,h)}{\partial h}\right]
\end{aligned}$$

We can apply 23 to cancel out terms and then substitute this expression into the second order condition and rearrange to find that we are left with

$$\frac{\partial^2 p_1(s_1,h)}{\partial s_1 \partial h}\frac{1}{d\sigma(s_1)} > -\frac{\phi'(h)}{1-\phi(h)}\frac{\partial p_1(s_1,h)}{\partial h} - \frac{u'(h)}{1+u(h)}\frac{\partial p_1(s_1,h)}{\partial h}$$

Adding and subtracting

$$\frac{\partial p_1(s_1,h)}{\partial s_1}\frac{1}{d\sigma(s_1)}\left[\frac{\phi'(h)}{1-\phi(h)} + \frac{u'(h)}{1+u(h)}\right]$$

to the second-order condition, applying 23, multiplying terms by $d\sigma(s_1)$, and then substituting for D. 5, we find the expression in equation D. 6.

We can now show that the single crossing condition will be satisfied whenever the inequality in equation D. 6 holds. We begin by examining the incentive compatibility constraints, which are now given by

$$\begin{aligned}
\phi(h) + (1-\phi(h))(p_1(s_1,h) - c_1) + u(h)[\phi(h) + (1-\phi(h))p_1(s_1,h)] \geq \\
\phi(h') + (1-\phi(h'))(p_1(s_1,h') - c_1) + u(h')[\phi(h') + (1-\phi(h')p_1(s_1,h')]
\end{aligned}$$

and

$$\phi(h')(1+u(h'))+(1-\phi(h'))(p_1(s_1',h')-c_1)+u(h')[\phi(h')+(1-\phi(h')p_1(s_1,h')] \geq$$
$$\phi(h)(1+u(h))+(1-\phi(h))(p_1(s_1',h)-c_1)+u(h)[\phi(h)+(1-\phi(h)p_1(s_1,h)]$$

where $s_1 > s_1'$ and $h > h'$. Adding the inequalities and following analogous steps to those in Proposition 2 of the main text, we find that the incentive compatibility constraints hold if

$$\frac{\partial^2 p_1(s_1,h)}{\partial s_1 \partial h} \geq \frac{\phi'(h)}{1-\phi(h)} \frac{p_1(s_1,h)}{\partial s_1} - \frac{u'(h)}{1+u(h)} \frac{\partial p_1(s_1,h)}{\partial s_1}$$

which is identical to the expression in D. 5 after substituting for the hazard rate. ■

## D.3 Augmenting the Value of the Good Being Bargained over

If both the Challenger and the Defender can enjoy the benefits of handicapping when they receive the good, then the Challenger's expected utility function remains D. 4. However, in this instance, the Defender is indifferent when $p_2(s_1,h) = c_2/(1+u(h))$. Moreover, the Defender's strategy again requires adjustments to account for these changes in the amount of handicapping required.

$$\frac{\phi'(h)}{1-\phi(h)} = -\frac{\partial p_1(s_1,h)}{\partial h} \frac{1+u(h)}{c_1+c_2} - \frac{u'(h)}{c_1+c_2} \left[ \frac{\phi(h)}{1-\phi(h)} + p_1(s_1,h) \right] \qquad \text{(D. 7)}$$

This equation has a similar interpretation to that in equation D. 5 and once again does not have a closed form solution.

The following proposition provides a characterization of the equilibrium. It shows that independent benefits make the sufficient condition on the cross partial more restrictive and a signaling equilibrium more difficult to achieve even when the Defender can also reap the benefits of handicapping when it receives the good.

**Proposition D. 3**

*If the cross partial on the lottery function satisfies*

$$\frac{\partial^2 p_1(s_1,h)}{\partial s_1 \partial h} > -\frac{\partial p_1(s_1,h)}{\partial s_1}\frac{\partial p_1(s_1,h)}{\partial h}\left[\frac{1+u(h)}{c_1+c_2}\right] \\ -\frac{u'(h)}{1+u(h)}\frac{\partial p_1(s_1,h)}{\partial s_1} + \frac{u'(h)\frac{\partial p_1(s_1,h)}{\partial s_1}}{c_1+c_2}\left[\frac{\phi(h)}{1-\phi(h)} + p_1(s_1,h)\right] \tag{D. 8}$$

then

(i) There exists a Perfect Bayesian Equilibrium where the Challenger plays

$$\sigma^*(s_1) \equiv \begin{cases} 0 & \text{if } p(s_1,0) \leq 1 - \frac{c_2}{1+u(h)} \\ \{h : p_2(s_1,h) = \frac{c_2}{1+u(h)}\} & \text{if } p(s_1,0) > 1 - \frac{c_2}{1+u(h)} \end{cases} \tag{D. 9}$$

and the Defender responds by playing D. 7 with $\phi(0) = 0$.

(ii) The uniqueness arguments in Proposition 2 parts (ii) and (iii) hold.

**Proof** The proof proceeds once more as in Proposition 2 in the main text: we only need show that D. 8 is necessary for the Challenger's expected utility function to be concave and $h$ and that this is sufficient for the Challenger's expected utility function to achieve single crossing.

As before D. 4 will be concave in $h$ whenever its second derivative with $h$ is negative. Note that the second-order condition will be identical to that in the proof of Proposition D.2 except that the square brackets in the first term will be given by $[c_1 + c_2]$ instead of $[c_1 + c_2[1 + u(h)]]$. With the same exception, we find an identical expression to that in the proof of Proposition D.2 when taking the derivative of $\phi'$as given in equation D. 7 with respect to $h$ while noting that in D. 7 $s_1$ is given by $\sigma^{-1}(h)$ with a similar exception. However, we cannot cancel terms as we did in Proposition D.2 because equation 23 no longer applies. Instead, note that $\sigma^*$ now implies that

$$\left[\frac{\partial p_1(s_1,h)}{\partial s_1}\frac{1}{d\sigma(s_1)} + \frac{\partial p_1(s_1,h)}{\partial h}\right][1+u(h)] - u'(h)[1-p_1(s_1,h)] = 0 \tag{D. 10}$$

Applying the above and then substituting the derivative of $\phi'$ into the second order condition we find that we are left with

$$
\begin{aligned}
-\frac{\partial^2 p_1(s_1,h)}{\partial s_1 \partial h}\frac{1}{d\sigma(s_1)}[1-\phi(h)][1+u(h)] - u'(h)[1-\phi(h)]\left[\frac{\partial p_1(s_1,h)}{\partial s_1}\frac{1}{d\sigma(s_1)} + \frac{\partial p_1(s_1,h)}{\partial h}\right] \\
-\phi'(h)[1+u(h)]\frac{\partial p_1(s_1,h)}{\partial h} + \phi'(h)u'(h)[1-p_1(s_1,h)] + u'(h)[1-\phi(h)]\frac{\partial p_1(s_1,h)}{\partial h} < 0
\end{aligned}
$$

We can cancel the last term and divide throughout by $[1+u(h)][1-\phi(h)]$ to find

$$
\begin{aligned}
-\frac{\partial^2 p_1(s_1,h)}{\partial s_1 \partial h}\frac{1}{d\sigma(s_1)} - \frac{u'(h)}{1+u(h)}\frac{\partial p_1(s_1,h)}{\partial s_1}\frac{1}{d\sigma(s_1)} \\
-\frac{\phi'(h)}{1-\phi(h)}\left[\frac{\partial p_1(s_1,h)}{\partial h} + \frac{u'(h)}{1+u(h)}[1-p_1(s_1,h)]\right] < 0
\end{aligned}
$$

Applying D. 10 to the terms in the square brackets, multiplying by $d\sigma(s_1)$ and rearranging we find that we are once again left with

$$
\frac{\partial^2 p_1(s_1,h)}{\partial s_1 \partial h} > \frac{\phi'(h)}{1-\phi(h)}\frac{\partial p_1(s_1,h)}{\partial s_1} - \frac{u'(h)}{1+u(h)}\frac{\partial p_1(s_1,h)}{\partial s_1}
$$

Substituting in for the hazard rate from equation D. 7, we find that the Challenger's expected utility function will be concave whenever D. 8 is satisfied.

To show that the single crossing condition will be satisfied whenever the inequality in equation D. 8 holds, we only need note that the fact that the expected utility function is identical to the case when the good was only being augmented for the Challenger. This implies that the incentive compatibility constraints are identical and the calculations conducted identical to those in Proposition D.2. with the sole difference that in the last step equation D. 7 is substituted in instead of equation D. 5.

The same arguments that established that these results are sufficient to guarantee the existence of a Perfect Bayesian Equilibrium along with those establishing uniqueness, including those involving D1, are the same as those used in Proposition 2 in the main text.■

## D.4 Generating Guaranteed Benefits

If the Challenger reaps benefits from handicaps independently of the outcome of the conflict, then the Challenger's expected utility function is given by

$$u(h) + \phi(h) + (1 - \phi(h))(p_1(s_1, h) - c_1) \tag{D. 11}$$

The Defender is once more indifferent when $p_2(s_1, h) = c_2$. This implies that the Challenger can once again continue to separate by playing the same strategy $\sigma_h^*$ as in equation 11 in the main text. However, the Defender's strategy must again change to ensure that the Challenger will find $\sigma^*$ incentive compatible. Now the Challenger will handicap itself up to the point where the following equation holds

$$\frac{\phi'(h)}{1 - \phi(h)} = -\frac{\frac{\partial p_1(s_1,h)}{\partial h}}{c_1 + c_2} - \frac{1}{1 - \phi(h)} \frac{u'(h)}{c_1 + c_2} \tag{D. 12}$$

This equation has a similar interpretation to those in the previous sections, except that the cost of handicapping is now independent of the benefit. The Defender's strategy does not have a closed form solution.

The following proposition provides a characterization of the equilibrium. As in the previous cases, it shows that the independent benefit makes the sufficient condition on the cross partial more restrictive and an equilibrium with signaling more difficult to achieve.

**Proposition D. 4**

*If the cross-partial on the lottery function satisfies*

$$\frac{\partial^2 p_1(s_1, h)}{\partial s_1 \partial h} > -\frac{\frac{\partial p_1(s_1,h)}{\partial s_1} \frac{\partial p_1(s_1,h)}{\partial h}}{c_1 + c_2} - \frac{d\sigma(s_1) u'(h) \frac{\partial p_1(s_1,h)}{\partial h}}{c_1 + c_2} \frac{1}{1 - \phi(h)} \tag{D. 13}$$

*then*

*(i) There exists a Perfect Bayesian Equilibrium where the Challenger plays $\sigma_h^*$ as in equa-*

*tion 11 in the main text and the Defender responds by playing equation D. 12 with* $\phi(0) = 0$.

(ii) *The uniqueness arguments in Proposition 2 parts (ii) and (iii) hold.*

**Proof:** The proof proceeds as it in Proposition 2 in the main text: I show that the condition D. 13 is necessary for the Challenger's expected utility function to be concave in $h$ and that this is sufficient for the single-crossing property to be attained. The remaining arguments are identical to those in the main text.

As before, D. 11 will be concave in $h$ whenever its second derivative with respect to $h$ is negative

$$(c_1 + c_2)\phi''(h) < 2\phi'(h)\frac{\partial p_1(s_1, h)}{\partial h} - (1 - \phi(h))\frac{\partial^2 p_1(s_1, h)}{\partial^2 h} - u''(h)$$

Again, we need learn more about $\phi''(h)$ to determine whether this inequality holds. Therefore, we take the derivative of $\phi'(h)$ as given in D. 12 with respect to $h$ while noting that in D. 12 $s_1$ is given by $\sigma^{-1}(h)$. This leaves us with

$$(c_1 + c_2)\phi''(h) = -(1 - \phi(h))\left[\frac{\partial^2 p_1(s_1, h)}{\partial^2 h} + \frac{\frac{\partial^2 p_1(s_1,h)}{\partial s_1 \partial h}}{d\sigma(s_1)}\right] + \phi'(h)\frac{\partial p_1(s_1, h)}{\partial h} - u''(h)$$

Then substituting this value into the Challenger's second second order condition, applying the identity found in equation 23, rearranging and substituting in for the hazard rate we find equation D. 13.

We can now show that the single crossing condition will be satisfied whenever the inequality in equation D. 13 holds. We begin by examining the incentive compatibility constraints which are now given by

$$u(h) + \phi(h) + (1 - \phi(h))(p_1(s_1, h) - c_1) \geq u(h')\phi(h') + (1 - \phi(h'))(p_1(s_1, h') - c_1)$$

$$u(h) + \phi(h') + (1 - \phi(h'))(p_1(s_1', h') - c_1) \geq u(h) + \phi(h) + (1 - \phi(h))(p_1(s_1', h) - c_1)$$

where $s_1 > s_1'$ and $h > h'$. Adding the inequalities and following similar steps as in Proposition 2 in the main text, we find that that the incentive compatibility constraints hold if

$$\frac{\partial^2 p_1(s_1,h)}{\partial s_1 \partial h} \geq \frac{\phi'(h)}{1-\phi(h)} \frac{p_1(s_1,h)}{\partial h} \tag{D. 14}$$

Then substituting in for the hazard rate we again find the equation given in D. 13.

The same arguments that established that these results are sufficient to guarantee the existence of a Perfect Bayesian Equilibrium along with those establishing uniqueness, including those involving D1, are the same as those used in Proposition2 in the main text. ■

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